MAP 4175 / 5177 Name:________________________________

Test 8 Date: October 23, 2019

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. Given $\mu_x = 0.02$, $\mu_y = 0.03$, and $\mu_{\overline{xy}} = 0.01$, determine ${}_{10}q_{xy}$

$\mu_x = 0.02 \;\Rightarrow\; {}_{10}p_x = e^{-0.2}$

$\mu_y = 0.03 \;\Rightarrow\; {}_{10}p_y = e^{-0.3}$

$\mu_{\overline{xy}} = 0.01 \;\Rightarrow\; {}_{10}p_{\overline{xy}} = e^{-0.1}$

${}_{10}p_{xy} = e^{-0.2} + e^{-0.3} - e^{-0.1} \approx 0.65471$

$$\therefore\; {}_{10}q_{xy} = 1 - {}_{10}p_{xy} \approx 0.34529$$

2. Given $\mu_x = 0.10$, determine $\overset{o}{e}_{x:\overline{10}|}$.

$$\overset{o}{e}_{x:\overline{10}|} = \int_0^{10} {}_tp_x \, dt = \int_0^{10} e^{-0.1t} \, dt = \frac{1}{0.1} e^{-0.1t} |_{10}^{0} = 10(1 - e^{-1}) \approx 6.32121$$

3. Given ${}_tp_{\overline{xy}} = (1.04)^{-t}$, determine $e_{\overline{xy}:\overline{20}|}$

$$e_{\overline{xy}:\overline{20}|} = \sum_{k=1}^{20} {}_kp_{\overline{xy}} = 1.04^{-1} + 1.04^{-2} + \cdots 1.04^{-20} = a_{\overline{20}|0.04} \approx 13.59033$$

For Numbers 4 and 5, assume lives are independent.

4. For two lives, both age 30, determine the probability that the first death occurs within 15 years using the SULT mortality for each life.

We seek the value of ${}_{15}q_{30:30}$

$${}_{15}p_{30:30} = {}_{15}p_{30} \cdot {}_{15}p_{30} = ({}_{15}p_{30})^2 = \left(\frac{l_{45}}{l_{30}}\right)^2 \approx 0.98614$$

$$\therefore\ {}_{15}q_{30:30} = 1 - {}_{15}p_{30:30} \approx 0.01386$$

5. For two lives, ages 25 and 30, determine the probability that the latter death occurs between 5 and 15 years from now using $DML(\omega = 100)$ mortality for each life.

We seek the value of ${}_{5|10}q_{\overline{25:30}}$

$${}_{5|10}q_{\overline{25:30}} = {}_{15}q_{\overline{25:30}} - {}_{5}q_{\overline{25:30}} = {}_{15}q_{25} \cdot {}_{15}q_{30} - {}_{5}q_{25} \cdot {}_{5}q_{30}$$

$${}_{15}q_{25} = \frac{15}{75}$$

$${}_{15}q_{30} = \frac{15}{70}$$

$${}_{5}q_{25} = \frac{5}{75}$$

$${}_{5}q_{30} = \frac{5}{70}$$

$$\therefore\ {}_{5|10}q_{\overline{25:30}} = \frac{15}{75} \cdot \frac{15}{70} - \frac{5}{75} \cdot \frac{5}{70} = \frac{200}{5250} = \frac{4}{105}$$