MAP 4175 / 5177 Name:______________
Quiz 9 Date: October 28, 2015

Show all work for full credit, and use correct notation. Simplify answers completely.

1. Given a triple decrement model with $p_x'^{(1)} = 0.9$, $p_x'^{(2)} = 0.8$, and $p_x'^{(3)} = 0.7$ determine $q_x^{(2)}$ using a constant force assumption for each decrement.

$p_x^{(\tau)} = .9(.8)(.7) = .504$

$q_x^{(\tau)} = 1 - .504 = .496$

$$CF \Rightarrow {}_tp_x'^{(2)} = \left[{}_tp_x^{(\tau)}\right]^{\left(q_x^{(2)}/q_x^{(\tau)}\right)}$$

$$t=1: \quad .8 = (.504)^{\left(q_x^{(2)}/.496\right)} \Rightarrow q_x^{(2)} \doteq .1615$$

or, more directly,

$$q_x^{(2)} = \int_0^1 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(2)}\,dt \overset{CF}{=} \int_0^1 e^{-\mu^{(\tau)}\cdot t}(\mu^{(2)})\,dt = \frac{\mu^{(2)}}{\mu^{(\tau)}}\left(1 - e^{-\mu^{(\tau)}}\right)$$

$$\left.\begin{aligned} \mu^{(2)} &= -\ln(.8) \\ \mu^{(\tau)} &= -\ln(.504) \end{aligned}\right\} \Rightarrow q_x^{(2)} = \frac{\ln(.8)}{\ln(.504)}(1 - .504) \doteq .1615$$

2. For a double decrement model where each decrement is uniformly distributed in the double decrement table, given $q_x'^{(1)} = 0.05$ and $q_x'^{(2)} = 0.10$, determine ${}_{0.5}p_x'^{(1)}$.

$$\text{MUDD}: \quad {}_tp_x'^{(1)} = \left[{}_tp_x^{(\tau)}\right]^{\left(q_x^{(1)}/q_x^{(\tau)}\right)}$$

First use $t=1$ to get $q_x^{(1)}$: $\quad p_x^{(\tau)} = (.95)(.9) = .855$

$q_x^{(\tau)} = .145$

$$.95 = (.855)^{\left(q_x^{(1)}/.145\right)}$$

$$\Rightarrow q_x^{(1)}/.145 = \frac{\ln(.95)}{\ln(.855)}$$

Now use $t = .5$, noting that ${}_{.5}q_x^{(\tau)} \overset{MUDD}{=} .5\,q_x^{(\tau)} = .0725$, and so

$${}_{.5}p_x^{(\tau)} = 1 - .0725 = .9275$$

$$\therefore \quad {}_{.5}p_x'^{(1)} = (.9275)^{\left[\ln(.95)/\ln(.855)\right]} \doteq .9757$$

3. Given a double decrement model where decrement 1 has $\mu_x^{(1)} = 0.01$ and decrement 2 is DML(100) in the associated single decrement table, determine ${}_{10}q_{75}^{(1)}$.

$${}_{10}q_{75}^{(1)} = {}_{10}q_{75}^{(\tau)} - {}_{10}q_{75}^{(2)} \qquad {}_{10}p_{75}^{(\tau)} = {}_{10}p_{75}^{\prime(1)} \cdot {}_{10}p_{75}^{\prime(2)} = \left(e^{-10(.01)}\right)\left(\frac{100-75-10}{100-75}\right) = .6\,e^{-.1}$$

$$\therefore {}_{10}q_{75}^{(\tau)} = 1 - .6e^{-.1}$$

$${}_{10}q_{75}^{(2)} = \int_0^{10} {}_tp_{75}^{\prime(1)} \cdot \underbrace{{}_tp_{75}^{\prime(2)} \cdot \mu_{75+t}^{(2)}}\, dt = \underbrace{q_{75}^{\prime(2)}} \int_0^{10} e^{-.01t}\, dt$$

$$= \underbrace{\frac{1}{25}} \cdot \frac{1}{.01} e^{-.01t}\Big|_{10}^{0} = 4(1-e^{-.1})$$

$$\therefore {}_{10}q_{75}^{(1)} = [1-.6e^{-.1}] - [4(1-e^{-.1})] = 3.4e^{-.1} - 3 = .0764$$

4. For a double decrement model where each decrement is uniformly distributed in its associated single decrement table, given $q_x^{\prime(1)} = 0.05$ and $q_x^{\prime(2)} = 0.10$, determine

(a) ${}_{0.5}q_x^{(2)}$ $$= \int_0^{0.5} \underline{{}_tp_x^{\prime(1)}} \cdot \underline{{}_tp_x^{\prime(2)} \cdot \mu_{x+t}^{(2)}}\, dt = \int_0^{0.5} \underline{(1-.05t)} \cdot \underline{(0.10)}\, dt$$

$$= 0.10\left[0.5 - (.025t^2)\Big|_0^{0.5}\right]$$

$$= 0.10\left[0.5 - .025(.5)^2\right] = .049375$$

(b) ${}_{0.5}q_x^{\prime(2)}$ $\underline{\text{SUDD}}$ $.5\, q_x^{\prime(2)} = .5(.10) = .05$