MAP 4175 / 5177 Name:_________________________________

Quiz 9 Date: November 9, 2016

Show all work for full credit, and use correct notation. Simplify answers completely.

1. Given a triple decrement model with $\mu_x^{(1)} = 1$, $\mu_x^{(2)} = 2$, and $\mu_x^{(3)} = 3$ determine ${}_{0.5}q_x^{(2)}$.

$$ {}_{.5}q_x^{(2)} = \int_0^{0.5} {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(2)}\,dt \qquad {}_tp_x^{(\tau)} = {}_tp_x'^{(1)} \cdot {}_tp_x'^{(2)} \cdot {}_tp_x'^{(3)} = e^{-t} \cdot e^{-2t} \cdot e^{-3t} = e^{-6t} $$

$$ = \int_0^{0.5} e^{-6t} \cdot 2\,dt = \frac{2}{6} e^{-6t}\Big|_{.5}^{0} = \frac{1}{3}(1 - e^{-3}) $$

(OR) since $\mu_x^{(2)} = \frac{1}{3}\mu_x^{(\tau)}$,

$$ {}_{.5}q_x^{(2)} = \frac{1}{3} \cdot {}_{.5}q_x^{(\tau)} = \frac{1}{3}(1 - {}_{.5}p_x^{(\tau)}) = \frac{1}{3}(1 - e^{-3}) $$

2. For a double decrement model where each decrement is uniformly distributed in the double decrement table, given $q_x'^{(1)} = 0.20$ and $q_x'^{(2)} = 0.40$, determine ${}_{0.25}p_x'^{(1)}$.

$$ {}_{.25}p_x'^{(1)} = \left[{}_{.25}p_x^{(\tau)}\right]^{\left(q_x^{(1)}/q_x^{(\tau)}\right)} $$

$$ p_x^{(\tau)} = p_x'^{(1)} \cdot p_x'^{(2)} = (.8)(.6) = .48 \Rightarrow q_x^{(\tau)} = .52 $$

$$ \text{MUDD} \Rightarrow {}_{.25}q_x^{(\tau)} = .25\left(q_x^{(\tau)}\right) = .25(.52) = .13 $$

$$ \therefore\ {}_{.25}p_x^{(\tau)} = .87 $$

need $q_x^{(1)}$: Use $p_x'^{(1)} = \left[p_x^{(\tau)}\right]^{\left(q_x^{(1)}/q_x^{(\tau)}\right)}$

$$ \Rightarrow .8 = (.48)^{\left(q_x^{(1)}/q_x^{(\tau)}\right)} \Rightarrow \frac{q_x^{(1)}}{q_x^{(\tau)}} = \frac{\ln(.8)}{\ln(.48)} $$

$$ \therefore\ {}_{.25}p_x'^{(1)} = (.87)^{\left(\frac{\ln(.8)}{\ln(.48)}\right)} = .9585\cdots $$

3. Given a double decrement model where decrement 1 has $\mu_x^{(1)} = 0.01$ and decrement 2 is DML(100) in the associated single decrement table, determine ${}_{10}q_{75}^{(1)}$.

Since (2) is DML(100) in ASDT, use ${}_{10}q_{75}^{(1)} = {}_{10}q_{75}^{(\tau)} - {}_{10}q_{75}^{(2)}$

$${}_{10}q_{75}^{(\tau)} = 1 - {}_{10}p_{75}^{\prime(1)} \cdot {}_{10}p_{75}^{\prime(2)} = 1 - e^{-10(.01)} \cdot \frac{15}{25} = 1 - .6e^{-.1}$$

$${}_{10}q_{75}^{(2)} = \int_0^{10} \underline{{}_tp_{75}^{\prime(1)}} \cdot \underline{{}_tp_{75}^{\prime(2)} \cdot \mu_{75+t}^{(2)}}\, dt = \int_0^{10} \underline{e^{-.01t}} \cdot \underline{q_{75}^{\prime(2)}}\, dt$$

$$= \frac{1}{25} \cdot \frac{1}{.01} e^{-.01t}\Big|_{10}^{0} = 4(1 - e^{-.1})$$

$$\therefore\ {}_{10}q_{75}^{(1)} = 1 - .6e^{-.1} - 4(1 - e^{-.1}) = 3.4e^{-.1} - 3 = .0764\ldots$$

4. For a double decrement model where $\mu_{x+t}^{(1)} = 2t$, and decrement 2 is MOY, you are given $q_x^{(2)} = 0.3$. Determine $q_x^{\prime(2)}$.

$$q_x^{(2)} = {}_{.5}p_x^{\prime(1)} \cdot q_x^{\prime(2)} \qquad {}_{.5}p_x^{\prime(1)} = e^{-\int_0^{.5} 2t\,dt}$$

$$= e^{-(.5^2 - 0)} = e^{-.25}$$

$$\therefore\ 0.3 = e^{-.25} \cdot q_x^{\prime(2)} \Rightarrow q_x^{\prime(2)} = .3e^{.25} = .3852\ldots$$

5. For a triple decrement model where decrements 1 and 2 are uniformly distributed in their associated single decrement tables, and decrement 3 is EOY, given $q_x^{(1)} = 0.1$, $q_x^{(2)} = 0.2$, and $q_x^{(3)} = 0.3$, determine ${}_{0.5}q_x'^{(2)}$.

Since (3) is EOY, we can ignore it.

$$q_x^{(1)} \stackrel{\text{SUDD}}{=} q_x'^{(1)}\left[1 - \frac{q_x'^{(2)}}{2}\right]$$

$$q_x^{(2)} \stackrel{\text{SUDD}}{=} q_x'^{(2)}\left[1 - \frac{q_x'^{(1)}}{2}\right]$$

Note: $q_x^{(1)} = .1$ & $q_x^{(2)} = .2$

Let $q_x'^{(1)} = a$ & $q_x'^{(2)} = b$

$$\therefore\ .1 = a\left(1 - \frac{b}{2}\right)$$
$$.2 = b\left(1 - \frac{a}{2}\right)$$

$$\Rightarrow \quad (-1)\left(.1 = a - \frac{ab}{2}\right)$$
$$+ \quad .2 = b - \frac{ab}{2}$$
$$.1 = b - a \Rightarrow a = b - .1$$

$$\therefore\ .1 = (b - .1)\left(1 - \frac{b}{2}\right) = b - .5b^2 - .1 + .05b$$

$$\Rightarrow .5b^2 - 1.05b + .2 = 0$$

$$\Rightarrow b = \frac{1.05 \pm \sqrt{1.05^2 - 4(.5)(.2)}}{2(.5)} = .2118\cdots$$

$$b = q_x'^{(2)} = .2118\cdots$$

$$\Rightarrow {}_{.5}q_x'^{(2)} \stackrel{\text{SUDD}}{=} .5 \cdot q_x'^{(2)} = .5(.2118\cdots) = .1059\cdots$$