Show all work for full credit, and use correct notation. Simplify answers completely. Unless told or implied otherwise, assume all lives are independent. See reverse side for additional problems. Each problem is worth 10 points.

1. For two lives, ages 30 and 40, use the ILT to determine

   (a) the probability that the first one dies within 10 years

   \[
   _{10}q_{30:40} = 1 - _{10}p_{30:40}^{ind} = 1 - _{10}P_{30} \cdot _{10}P_{40} = 1 - \frac{L_{30}}{L_{30}} \cdot \frac{L_{50}}{L_{40}} = 1 - \frac{L_{50}}{L_{30}}
   \]

   \[
   \therefore _{10}q_{30:40} \stackrel{ILT}{\approx} 0.01154 \ldots
   \]

   (b) the probability that the last death occurs at least 10 years from now

   \[
   _{10}p_{30:40} = 1 - _{10}q_{30:40}^{ind} = 1 - _{10}q_{30} \cdot _{10}q_{40} = 1 - (1 - \frac{L_{30}}{L_{30}})(1 - \frac{L_{50}}{L_{40}})
   \]

   \[
   \therefore _{10}p_{30:40} \stackrel{ILT}{\approx} 0.999997 \ldots
   \]

2. You are given:

   (i) For 20-year old smokers, \( \mu = 0.02 \)

   (ii) 40-year old non-smoker mortality is uniformly distributed with terminal age \( \omega = 100 \).

   (iii) \( _{10}q_{20:40} = \int_{0}^{10} g(t) \, dt \) where (20) is a smoker and (40) is a non-smoker.

   Determine \( g(5) \).

   \[
   g(t) = t \cdot \frac{q_{40}}{L_{20}} \cdot \frac{p_{20} \cdot \mu_{20}(t)}{e^{\omega \cdot t}} = \frac{5}{60} \cdot e^{-0.1} \cdot (0.02)
   \]

   \[
   \therefore g(5) = \frac{5}{60} \cdot e^{-0.1} \cdot (0.02)
   \]

3. You are given:

   (i) Male mortality is \( DML(\omega = 100) \)

   (ii) Female mortality is \( DML(\omega = 110) \)

   If (50) is male and (60) is female, determine a simplified expression for \( tP_{50:60} \)

   \[
   tP_{50:60} \stackrel{ind}{=} tP_{50} \cdot tP_{60}^{f} = \left( \frac{50-t}{50} \right) \left( \frac{50-t}{50} \right) = \left( \frac{50-t}{50} \right)^2
   \]
4. You are given:

(i) For smokers, \( \mu_x = 0.04 \)
(ii) For non-smokers, \( \mu_x = 0.0003 \cdot (1.07)^x \)

Determine the difference \( 25q_{40:50} \cdot 25P_{q40} \) where (40) is a smoker and (50) is not.

\[
\Delta = 25q_{50} \cdot 25P_{q40} = e^{-0.04(25)} = e^{-1}
\]

\[
25q_{50} = 1 - 25P_{q50} = e^{-0.0003 \cdot 0.603 \cdot (1.07)^{75} \cdot \ln(1.07) \cdot (1.07^{75} - 1.07^{50})}
\]

\[
\Rightarrow 25P_{q50} = 0.5608\ldots
\]

\[
\Rightarrow 25q_{50} = 0.4391\ldots
\]

\[
\therefore \Delta = (0.4391\ldots) \cdot e^{-1} = 0.1615\ldots
\]

5. Given a male whose force of mortality is \( \mu^m = 0.3 \), and a female whose force of mortality is \( \mu^f = 0.1 \), determine the probability that the male dies second.

\[
\infty b_{m,f} = \infty b_{m,f} \cdot \frac{\mu^f}{\mu^m + \mu^f} = \infty b_{m,f} = \frac{0.1}{0.1 + 0.3} = \frac{1}{4}
\]

More directly,

\[
\infty b_{m,f} = \int_0^\infty t^f \cdot t^m \cdot \mu_m(t) \, dt = \int_0^\infty (1 - e^{-t}) \cdot e^{-3t} \cdot (1.3) \, dt
\]

\[
= 1.3 \int_0^\infty (e^{-3t} - e^{-4t}) \, dt
\]

\[
= 1.3 \left[ \frac{1}{3} \cdot e^{-3t} \bigg|_0^\infty - \frac{1}{4} \cdot e^{-4t} \bigg|_0^\infty \right]
\]

\[
= 1.3 \left[ \frac{1}{3} - \frac{1}{4} \right] = 1 - \frac{3}{4} = \frac{1}{4}
\]