MAP 4175 / 5177 Name: KEY

Quiz 9 Date: October 31, 2018

Show all work for full credit, and use correct notation. Simplify answers completely. Unless told or implied otherwise, assume all lives are independent. See reverse side for additional problems. Each problem is worth 10 points

1. For two lives, ages 30 and 40, use the ILT to determine

   (a) the probability that the first one dies within 10 years

   $${}_{10}q_{30:40} = 1 - {}_{10}p_{30:40} \overset{ind.}{=} 1 - {}_{10}p_{30} \cdot {}_{10}p_{40} = 1 - \frac{\ell_{40}}{\ell_{30}} \cdot \frac{\ell_{50}}{\ell_{40}} = 1 - \frac{\ell_{50}}{\ell_{30}}$$

   $$\therefore \; {}_{10}q_{30:40} \overset{ILT}{=} 0.01154\ldots$$

   (b) the probability that the last death occurs at least 10 years from now

   $${}_{10}p_{\overline{30:40}} = 1 - {}_{10}q_{\overline{30:40}} \overset{ind}{=} 1 - {}_{10}q_{30} \cdot {}_{10}q_{40} = 1 - \left(1 - \frac{\ell_{40}}{\ell_{30}}\right)\left(1 - \frac{\ell_{50}}{\ell_{40}}\right)$$

   $$\therefore \; {}_{10}p_{\overline{30:40}} \overset{ILT}{=} 0.99997\ldots$$

2. You are given:

   (i) For 20-year old smokers, $\mu = 0.02$

   (ii) 40-year old non-smoker mortality is uniformly distributed with terminal age $\omega = 100$.

   (iii) ${}_{10}q^{2}_{20:40} = \int_0^{10} g(t)\; dt$ where (20) is a smoker and (40) is a non-smoker.

   Determine $g(5)$.

   $$g(t) = \underbrace{{}_tq_{40}}_{= t/60} \cdot \underbrace{{}_tp_{20}}_{= e^{-.02t}} \cdot \underbrace{\mu_{20}(t)}_{= 0.02}$$

   $$\therefore \; g(5) = \frac{5}{60} \cdot e^{-0.1} \cdot (0.02)$$

3. You are given:

   (i) Male mortality is $DML(\omega = 100)$

   (ii) Female mortality is $DML(\omega = 110)$

   If (50) is male and (60) is female, determine a simplified expression for ${}_tp_{50:60}$

   $${}_tp_{50:60} \overset{ind}{=} {}_tp^{m}_{50} \cdot {}_tp^{f}_{60} = \left(\frac{50-t}{50}\right)\left(\frac{50-t}{50}\right) = \left(\frac{50-t}{50}\right)^2$$

4. You are given:

(i) For smokers, $\mu_x = 0.04$
(ii) For non-smokers, $\mu_x = 0.0003 \cdot (1.07)^x$

Determine the difference ${}_{25}q_{40:\overset{1}{50}} - {}_{25}q_{\overset{2}{40}:50}$ where (40) is a smoker and (50) is not.

$$\Delta = {}_{25}q_{50} \cdot {}_{25}p_{40}$$

$${}_{25}p_{40} \overset{S}{=} e^{-.04(25)} = e^{-1}$$

$${}_{25}q_{50} = 1 - {}_{25}p_{50}$$

$${}_{25}p_{50} \overset{NS}{=} e^{-\int_{50}^{75} .0003\,(1.07)^x\,dx} = e^{-\frac{.0003}{\ln(1.07)} \cdot (1.07^{75} - 1.07^{50})}$$

$$\Rightarrow {}_{25}p_{50} = 0.5608\ldots$$

$$\Rightarrow {}_{25}q_{50} = 0.4391\ldots$$

$$\therefore \Delta = (0.4391\ldots) \cdot e^{-1} = 0.1615\ldots$$

5. Given a male whose force of mortality is $\mu^m = 0.3$, and a female whose force of mortality is $\mu^f = 0.1$, determine $\underbrace{\text{the probability that the male dies second}}_{= {}_{\infty}q^{\,2}_{m:f}}$.

$${}_{\infty}q^{\,2}_{m:f} = {}_{\infty}q_{m:\overset{1}{f}} \overset{CF}{=} \frac{\mu^f}{\mu^f + \mu^m} = \frac{0.1}{0.1 + 0.3} = 1/4$$

More directly,

$${}_{\infty}q^{\,2}_{m:f} = \int_0^\infty {}_tq_f \cdot {}_tp_m \cdot \mu_m(t)\,dt$$

$$= \int_0^\infty (1 - e^{-.1t}) \cdot e^{-.3t} \cdot (.3)\,dt$$

$$= .3 \int_0^\infty (e^{-.3t} - e^{-.4t})\,dt$$

$$= .3\left[\frac{1}{.3} e^{-.3t}\Big|_\infty^0 - \frac{1}{.4} e^{-.4t}\Big|_\infty^0\right]$$

$$= .3\left[\frac{1}{.3} - \frac{1}{.4}\right] = 1 - \frac{3}{4} = \frac{1}{4}$$