MAP 4175 / 5177 Name: KEY

Test 9 (Take-Home – Due: In Class on November 5) Date: October 30, 2019

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. Assume all lives are independent of one another.

For numbers 1 and 2: Male mortality follows a $DML(\omega = 90)$ model and female mortality follows a $GDML(\alpha = 1.5, \omega = 100)$ model.

1. Determine ${}_{10}q_{20:30}$ where (20) is a female and (30) is a male.

$${}_{10}p_{20:30} = {}_{10}p_{20}^{f} \cdot {}_{10}p_{30}^{m} = \left(\frac{100-20-10}{100-20}\right)^{1.5} \cdot \left(\frac{90-30-10}{90-30}\right) = \left(\frac{7}{8}\right)^{1.5} \cdot \frac{5}{6}$$

$$\Rightarrow {}_{10}q_{20:30} = 1 - \left(\frac{7}{8}\right)^{1.5} \cdot \frac{5}{6} = 0.317\ldots$$

2. Determine $\overset{o}{e}_{\overline{20:30}}$ where (20) is a male and (30) is a female.

$$\overset{o}{e}{}_{20}^{m} = \frac{90-20}{2} = 35$$

$$\overset{o}{e}{}_{30}^{f} = \frac{100-30}{2.5} = 28$$

$${}_tp_{20:30} = {}_tp_{20}^{m} \cdot {}_tp_{30}^{f} = \left(\frac{70-t}{70}\right) \cdot \left(\frac{70-t}{70}\right)^{1.5} = \left(\frac{70-t}{70}\right)^{2.5}$$

$\hookrightarrow GDML(\omega - x = 70, \alpha = 2.5)$

$$\therefore \overset{o}{e}_{20:30} = \frac{70}{3.5} = 20$$

$$\therefore \overset{o}{e}_{\overline{20:30}} = 35 + 28 - 20 = 43$$

For numbers 3 – 5: Smoker mortality follows a constant force model with $\mu = 0.02$, whereas non-smoker mortality follows a $DML(\omega = 100)$ model.

3. Determine ${}_{10}p_{\overline{30:50}}$ where (30) is a smoker and (50) is a non-smoker.

$${}_{10}q_{\overline{30:50}} = {}_{10}q^{s}_{30} \cdot {}_{10}q^{ns}_{50} = (1-e^{-.02(10)}) \cdot \frac{10}{50} = \frac{1}{5} - \frac{1}{5}e^{-0.2}$$

$$\Rightarrow {}_{10}p_{\overline{30:50}} = 1 - \left(\frac{1}{5} - \frac{1}{5}e^{-0.2}\right) = \frac{4}{5} + \frac{1}{5}e^{-0.2} = 0.963\ldots$$

4. Determine ${}_{10}q^{\;1}_{30:50}$ where (30) is a smoker and (50) is a non-smoker.

$${}_{10}q_{30:\overset{1}{50}} = \int_0^{10} \underbrace{{}_tp^{s}_{30}}_{e^{-.02t}} \cdot \underbrace{{}_tp^{ns}_{50} \cdot \mu_{50+t}}_{=\frac{1}{50}}\, dt = \frac{1}{50}\int_0^{10} e^{-.02t}\, dt$$

$$= \frac{1}{50}\cdot\frac{1}{.02}\, e^{-.02t}\Big|_{10}^{0} = 1 - e^{-0.2}$$

$${}_{10}q_{30:50} = 1 - {}_{10}p^{s}_{30} \cdot {}_{10}p^{ns}_{50} = 1 - e^{-0.2}\cdot\frac{4}{5} = 1 - 0.8e^{-0.2}$$

$${}_{10}q^{\;1}_{30:50} = {}_{10}q_{30:50} - {}_{10}q_{30:\overset{1}{50}}$$

$$= (1 - 0.8e^{-0.2}) - (1 - e^{-0.2}) = 0.2e^{-0.2}$$

5. Determine $\overset{o}{e}_{\overline{30:40}:\overline{10}|}$ where both (30) and (40) are smokers.

$$\overset{o}{e}_{30:\overline{10}|} = \int_0^{10} e^{-.02t}\, dt = \frac{-1}{.02}e^{-.02t}\Big|_{10}^{0} = \cancel{e^{-0.2}}\ 50(1-e^{-0.2})$$

$$\overset{o}{e}_{40:\overline{10}|} = \overset{o}{e}_{30:\overline{10}|} = \cancel{e^{-0.2}}\ 50(1-e^{-0.2})$$

$$\overset{o}{e}_{30:40:\overline{10}|} = \int_0^{10} {}_tp_{30}\cdot {}_tp_{40}\, dt = \int_0^{10} e^{-.02t}\cdot e^{-.02t}\, dt$$

$$= \int_0^{10} e^{-.04t}\, dt = \frac{1}{.04}e^{-.04t}\Big|_{10}^{0} = 25(1-e^{-0.4})$$

$$\therefore\ \overset{o}{e}_{\overline{30:40}:\overline{10}|} = 50(1-e^{-0.2}) + 50(1-e^{-0.2}) - 25(1-e^{-0.4})$$

$$= 75 - 100e^{-0.2} + 25e^{-0.4} = 9.8849\ldots$$