MAP 4176 / 5178 Name:__________
Test 11 Date: April 11, 2018

Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. For a fully discrete whole life insurance of 150,000 issued to (40), you are given:

   (i) $\ddot{a}_{40} = 15$
   (ii) $\ddot{a}_{55} = 12$
   (iii) $d = 0.06$
   (iv) $p_{40} = 0.98$
   (v) The only expenses are 50 at the beginning of every year

$$\left.\begin{array}{l}\text{(i)}\\ \text{(ii)}\\ \text{(iii)}\end{array}\right\} \Rightarrow \begin{array}{l} A_{40} = 1 - d\cdot\ddot{a}_{40} = 0.1 \\ A_{55} = 1 - d\,\ddot{a}_{55} = 0.28 \end{array}$$

Determine

(a) (10 points) the gross premium using the equivalence principle, and the corresponding gross premium reserve at time $k = 15$

$$\Pi^g \cdot \ddot{a}_{40} = 150000\cdot A_{40} + 50\cdot\ddot{a}_{40} \Rightarrow \Pi^g = 1050$$

$$_{15}V^g = 150000\cdot A_{55} + 50\cdot\ddot{a}_{55} - 1050\,\ddot{a}_{55} = 30000$$

(b) (10 points) the expense premium and the corresponding expense premium reserve at time $k = 15$

$$\Pi^n = \frac{150000\,A_{40}}{\ddot{a}_{40}} = 1000 \Rightarrow \Pi^e = \Pi^g - \Pi^n = 50$$

$$_{15}V^e = 50\cdot\ddot{a}_{55} - \Pi^e\cdot\ddot{a}_{55} = 0$$

(c) (10 points) the full preliminary term reserve at time $k = 15$

$$_{15}V^{FPT} = {}_{14}V^n_{41} = 150000\left(1 - \frac{\ddot{a}_{55}}{\ddot{a}_{41}}\right)$$

$$\ddot{a}_{40} = 1 + v\,p_{40}\cdot\ddot{a}_{41} \Rightarrow \ddot{a}_{41} = 15.19756\ldots$$

$$\therefore\ {}_{15}V^{FPT} = 31,560$$

2. For a fully discrete insurance issued to (x) you are given:

   (i) the death benefit is 10000

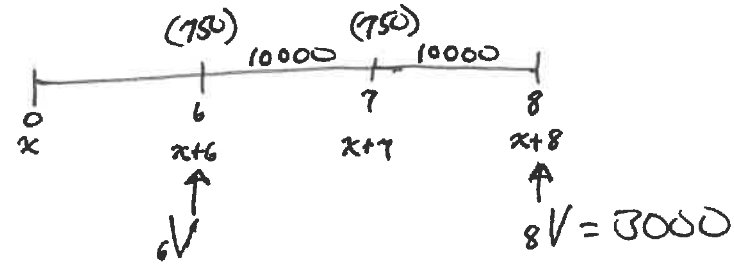
   (ii) the annual premium is 750

   (ii) $p_{x+6} = 0.95$ and $p_{x+7} = 0.90$

   (iii) $i = 4\%$

   (iii) ${}_8V = 3000$

   Determine ${}_6V$



$${}_6V = 10000\, v\, q_{x+6} + 10000\, v^2 \cdot p_{x+6} \cdot q_{x+7} - 750 - 750\, v\, p_{x+6} + {}_8V \cdot v^2 \cdot p_{x+6} \cdot p_{x+7}$$

$$\Rightarrow {}_6V = 2,295.488\ldots$$

3. For a fully discrete whole life insurance of 10000 issued (30), you are given:

   (i) the death benefit is paid at the end of the quarter of death

   (ii) premiums of 15 are paid at the beginning of each quarter

   (iii) $A_{40} = 0.15$

   (iv) $i = 0.05$

   Assuming a uniform distribution of deaths between integer ages, determine the reserve at time $k = 10$.

$${}_{10}V = 10000\, A_{40}^{(4)} - 4 \cdot 15 \cdot \ddot{a}_{40}^{(4)} = 10000\, A_{40}^{(4)} - 60 \cdot \ddot{a}_{40}^{(4)}$$

$$A_{40}^{(4)} = \frac{i}{i^{(4)}} \cdot A_{40} \qquad \left(1 + \frac{i^{(4)}}{4}\right)^4 = 1 + i = 1.05$$

$$\therefore A_{40}^{(4)} = 0.15278\ldots$$

$$\ddot{a}_{40}^{(4)} = \frac{1 - A_{40}^{(4)}}{d^{(4)}} \qquad \left(1 - \frac{d^{(4)}}{4}\right)^{-4} = 1 + i = 1.05$$

$$\therefore \ddot{a}_{40}^{(4)} = 17.4706\ldots$$

$$\therefore {}_{10}V = 479.602\ldots$$