MAP 4176 / 5178 Name:______________________________

Test 11 Date: April 3, 2019

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Unless implied or told otherwise, premiums are level.

1. Each of 5,000 independent lives age 20 purchases a fully discrete whole life insurance of 10,000. Using SULT actuarial assumptions and the normal approximation, determine the annual premium each pays such that the probability of a total loss is 5%. Note that the 95th percentile of the standard normal distribution is 1.645.

$T = \sum_1^{5000} \left({}_0L \right)_i$ where ${}_0L = 10000 \cdot Z_{20} - \pi \cdot \ddot{Y}_{20} = \left(10000 + \frac{\pi}{d}\right) \cdot Z_{20} - \frac{\pi}{d}$

We seek the value of π such that $\Pr(T > 0) = 0.05$. Since T is normal, $\Pr(T > 0) = \Pr\left(SND > \frac{-E[T]}{\sqrt{Var(T)}}\right)$, and so we seek the value of π such that $\frac{-E[T]}{\sqrt{Var(T)}}$ equals the 95th percentile of the standard normal distribution. So we seek the value of π such that $-E[T] = 1.645 \cdot \sqrt{Var(T)}$.

Since $E\left[{}_0L\right] = 10000 \cdot A_{20} - \pi \cdot \ddot{a}_{20} = 492.2 - 19.9664\pi$

and $Var\left({}_0L\right) = \left(10000 + \frac{\pi}{d}\right)^2 \cdot \left({}^2A_{20} - (A_{20})^2\right) = (10000 + 21\pi)^2 \cdot (0.00337 \cdots)$,

then $E[T] = 5000 \cdot (492.2 - 19.9664\pi)$

and $Var(T) = 5000 \cdot (10000 + 21\pi)^2 \cdot (0.00337 \cdots)$

$$\therefore -5000 \cdot (492.2 - 19.9664\pi) = 1.645 \cdot \sqrt{5000 \cdot (10000 + 21\pi)^2 \cdot (0.00337 \cdots)}$$

$$\therefore \pi = 25.36$$

2. A 4-year fully discrete term insurance issued to (40) has a death benefit of 1000 in each year. Annual premiums are 20. Using $d = 0.05$, determine the value of ${}_1L | K_{40} = 2$.

$$\left({}_1L | K_{40} = 2 \right) = 1000v^2 - 20 - 20v = 863.5$$

3. Given a fully continuous whole life insurance of 5000 issued (x), determine the annual premium rate such that there is a 90% probability that there is a gain at issue on the policy. Assume $CF(\mu = 0.01, \delta = 0.04)$ actuarial assumptions.

Since the probability of a gain at issue on the policy equals $\Pr\left({}_0L < 0\right)$, we seek the value of π such that $\Pr\left({}_0L < 0\right) = 0.90$.

$$ {}_0L = 5000 \cdot \bar{Z}_x - \pi \cdot \bar{Y}_x = \left(5000 + \tfrac{\pi}{\delta}\right) \cdot \bar{Z}_x - \tfrac{\pi}{\delta} = (5000 + 25\pi) \cdot v^T - 25\pi $$

$$ \therefore \Pr\left({}_0L < 0\right) = \Pr\left(e^{-\delta T} < \frac{25\pi}{5000 + 25\pi}\right) = \Pr\left(T > \frac{ln\left(\frac{25\pi}{5000 + 25\pi}\right)}{-\delta}\right) $$

$$ = e^{-\mu \frac{ln\left(\frac{25\pi}{5000+25\pi}\right)}{-\delta}} = \left(\frac{25\pi}{5000 + 25\pi}\right)^{\frac{\mu}{\delta}} = \sqrt[4]{\frac{25\pi}{5000 + 25\pi}} $$

$$ \therefore \sqrt[4]{\frac{25\pi}{5000 + 25\pi}} = 0.9 \quad \Rightarrow \quad \pi = 382 $$

4. A fully continuous whole life insurance of 3000 issued to (x) has an annual premium rate of 80 that's payable for a maximum of 10 years. Using $CF(\mu = .01, \delta = .04)$, Determine $Var\left({}_{10}L\right)$.

Since there are no more future premiums at time $t = 10$, then ${}_{10}L = 3000 \cdot \bar{Z}_{x+10}$

So $Var\left({}_{10}L\right) = 3000^2 \cdot Var(\bar{Z}_{x+10}) = 3000^2 \cdot \left({}^2\bar{A}_{x+10} - (\bar{A}_{x+10})^2\right)$

$$ = 3000^2 \cdot \left(\frac{\mu}{\mu + 2\delta} - \left(\frac{\mu}{\mu + \delta}\right)^2\right) = 640{,}000 $$

5. For a fully discrete whole life insurance of 10,000 issued to (20), annual premiums are determined by the equivalence principle. Determine the time 5 reserve, ${}_5V$. Use the SULT for all calculations.

$$ {}_5L = 10000 \cdot Z_{25} - \pi \cdot \ddot{Y}_{25} \quad \Rightarrow \quad {}_5V = E[\,{}_5L] = 10000 \cdot A_{25} - \pi \cdot \ddot{a}_{25} $$

$$ \pi = \frac{10000 \cdot A_{20}}{\ddot{a}_{20}} $$

$$ \therefore \; {}_5V = 129 $$