MAP 4176 / 5178 Name:

Test 12 (Due in Class on April 19.) Date: April 18, 2018

Show all work for full credit, and use correct notation. Simplify answers completely. See other side of each page for additional problems.

1. For a fully discrete whole life insurance of 1000 on (x), you are given:

   (i) Death is the only decrement

   (ii) The annual gross premium is 100

   (iii) Expenses are 60% of gross premium for the first year, 10% thereafter, payable at BOY

   (iv) $i = 0.05$

   (v) $q_x = 0.030$ and $q_{x+1} = 0.035$

   Determine the asset share at the end of the second year.

The idea is the same as with gross premium reserves; i.e. replace ${}_kV$ by AS_k

2-year recursion

$$AS_0 = 1000 v q_x + 1000 v^2 \cdot {}_{1|}q_x + 60 + 10 v p_x - 100 - 100 v p_x + AS_2 \cdot v^2 \cdot {}_2p_x$$

$$\Rightarrow 0 = \frac{1000}{1.05}(.03) + \frac{1000}{1.05^2} \cdot (.97)(.035) + 60 + \frac{10}{1.05}(.97) - 100 - \frac{100}{1.05}(.97) + AS_2 \cdot \frac{(.97)(.965)}{1.05^2}$$

$$\Rightarrow AS_2 = 75.118\ldots$$

2. For a fully discrete whole life insurance on (x), you are given:

(i) The death benefit is 10,000.

(ii) The withdrawal benefit for year 5, paid at EOY, is 450.

(iii) The annual gross premium is 300

(iv) Expenses during year 5 are 10% of gross premium

(v) $i = 6\%$

(vi) $q_{x+4}^{(d)} = .02$ and $q_{x+4}^{(w)} = .10$

(vii) Reserves are set as follows: ${}_4V = 400$ and ${}_5V = 500$

Determine Pr_5, the profit emerging at the end of year 5 per policy in force at the beginning of the year.

$$Pr_5 = ({}_4V + \pi - e)(1+i) - F \cdot q_{x+4}^{(d)} - W \cdot q_{x+4}^{(w)} - {}_5V \cdot p_{x+4}^{(\tau)}$$

$$= (400 + 300 - 30)(1.06) - 10000(.02) - 450(.1) - 500 \cdot (1 - .02 - .1)$$

$$= 25.2$$

3. For a fully discrete whole life insurance on (x), you are given:

(i) The death benefit is 10,000.

(ii) The withdrawal benefit for year 10, paid at EOY, is W.

(iii) The annual gross premium is π.

(iv) Expenses during year 10 total e, payable at the beginning of the year.

(v) $q_{x+9}^{(w)} = 0.10$

(vi) $q_{x+9}^{(d)} = 0.020$ is the actual value, and $q_{x+9}^{(d)} = 0.015$ is the expected value.

(vii) ${}_{10}V = 1500$

Determine the gain/loss due to mortality for year 10 per policy in force at the beginning of the year.

$$Pr_{10} = \underbrace{({}_9V + \pi - e)(1+i)}_{=X} - 10000\, q_{x+9}^{(d)} - \underbrace{W \cdot q_{x+9}^{(w)}}_{=Y} - \overbrace{{}_{10}V}^{=1500} \cdot p_{x+9}^{(\tau)}$$

$$\therefore Pr_{10} = X - 10000\, q_{x+9}^{(d)} - Y - 1500\, p_{x+9}^{(\tau)}$$

$$p_{x+9}^{(\tau)} = 1 - q_{x+9}^{(d)} - q_{x+9}^{(w)}$$

$$G^m = \{Pr_{10}\,(A-m)\} - [Pr_{10}\,(E-m)]$$

$$= \{X - 10000(.02) - Y - 1500(\overbrace{1-.02-.1}^{=.88})\}$$

$$- [X - 10000(.015) - Y - 1500(\overbrace{1-.015-.1}^{=.885})]$$

$$= \{X - Y - 1520\} - [X - Y - 1477.5]$$

$$\therefore G^m = -42.5$$

4. For a fully discrete 5-year term insurance on (x), you are given:

(i) The annual gross premium is 1200.

(ii) The profit vector is $Pr = (-900, 265, 265, 265, 265, 265)$.

(iii) Mortality follows a constant force model with $\mu = -\ln(0.9)$. $\Rightarrow {}_nP_x = .9^n$

(iv) The hurdle rate is $i = 0.05$.

$Pr_0 = -900 \quad \Pi_0 = Pr_0 = -900$

$Pr_1 = 265 \quad \Pi_1 = Pr_1 = 265$

$Pr_2 = 265 \quad \Pi_2 = Pr_2 \cdot (P_x) = 265(.9)$

$Pr_3 = 265 \quad \Pi_3 = Pr_3 ({}_2P_x) = 265(.9)^2$

$Pr_4 = 265 \quad \Pi_4 = Pr_4 ({}_3P_x) = 265(.9)^3$

$Pr_5 = 265 \quad \Pi_5 = Pr_5 ({}_4P_x) = 265(.9)^4$

Determine

(a) (10 points) $NPV(3)$

(b) (10 points) The profit margin for this policy.

(a) $NPV(3) = \Pi_0 + \Pi_1 v + \Pi_2 v^2 + \Pi_3 v^3$

$$= -900 + \frac{265}{1.05} + \frac{265(.9)}{1.05^2} + \frac{265(.9)^2}{1.05^3}$$

$\therefore NPV(3) = -245.869\cdots$

(b) $PM = \dfrac{NPV}{APV(\text{premiums})}$

$NPV = \Pi_0 + \Pi_1 v + \Pi_2 v^2 + \Pi_3 v^3 + \Pi_4 v^4 + \Pi_5 v^5 = 49.292\ldots$

$APV(\text{Premiums}) = 1200 \cdot \ddot{a}_{x:\overline{5}|} = 1200(1 + vP_x + v^2 \cdot {}_2P_x + v^3 \cdot {}_3P_x + v^4 \cdot {}_4P_x)$

$$= 1200\left(1 + \frac{.9}{1.05} + \frac{.9^2}{1.05^2} + \frac{.9^3}{1.05^3} + \frac{.9^4}{1.05^4}\right) = 4513.619\ldots$$

$\therefore PM = 0.01092\cdots$