1. For a fully discrete whole life insurance of 1000 on \((x)\), you are given:

(i) Death is the only decrement

(ii) The annual gross premium is 100

(iii) Expenses are 60% of gross premium for the first year, 10% thereafter, payable at BOY

(iv) \(i = 0.05\)

(v) \(q_x = 0.030\) and \(q_{x+1} = 0.035\)

Determine the asset share at the end of the second year.

The idea is the same as with gross premium reserves, i.e. replace \(kV\) by \(\text{AS}_x\)

2-year recursion

\[
\text{AS}_0 = 1000v b_x + 1000v^2 b_x + 60 + 10v p_x - 100 - 100v^2 p_x + \text{AS}_2 \cdot v^2 p_x
\]

\[
\Rightarrow 0 = \frac{1000}{1.05} (0.03) + \frac{1000}{1.05^2} \cdot (0.97) (0.035) + 60 + \frac{10}{1.05} (0.97) - 100 - \frac{1000}{1.05^2} (0.97) + \text{AS}_2 \cdot \frac{0.97 \cdot 0.035}{1.05^2}
\]

\[
\Rightarrow \text{AS}_2 = 75.118 \ldots
\]
2. For a fully discrete whole life insurance on \((x)\), you are given:

(i) The death benefit is 10,000.

(ii) The withdrawal benefit for year 5, paid at EOY, is 450.

(iii) The annual gross premium is 300

(iv) Expenses during year 5 are 10\% of gross premium

(v) \(i = 6\%\)

(vi) \(q_x^{(d)} = 0.02\) and \(q_x^{(w)} = 0.10\)

(vii) Reserves are set as follows: \(\,_4V = 400\) and \(\,_5V = 500\)

Determine \(Pr_5\), the profit emerging at the end of year 5 per policy in force at the beginning of the year.

\[
Pr_5 = (\,_4V + \pi - e)(1+i) - F \cdot q_x^{(w)} - W \cdot q_x^{(w)} - \,_5V \cdot P_{x+4}^{(e)}
\]

\[
= (400 + 300 - 30)(1.06) - 10000(0.02) - 450(0.1) - 500 \cdot (1 - 0.02 - 1)
\]

\[
= 25.2
\]
3. For a fully discrete whole life insurance on \( (x) \), you are given:

(i) The death benefit is 10,000.

(ii) The withdrawal benefit for year 10, paid at EOY, is \( W \).

(iii) The annual gross premium is \( \pi \).

(iv) Expenses during year 10 total \( e \), payable at the beginning of the year.

(v) \( q_{x+9}^{(w)} = 0.10 \)

(vi) \( q_{x+9}^{(d)} = 0.020 \) is the actual value, and \( q_{x+9}^{(d)} = 0.015 \) is the expected value.

(vii) \( _{10}V = 1500 \)

Determine the gain/loss due to mortality for year 10 per policy in force at the beginning of the year.

\[
\begin{align*}
Pr_{10} &= \frac{(qV + \pi - e)(1+i)}{10000} - W \cdot q_{x+9}^{(w)} - _{10}V \cdot P_{x+9}^{(c)} \\
&= \frac{1500}{2} = Y
\end{align*}
\]

\[
\therefore \quad Pr_{10} = X - 10000 \cdot g_{x+9}^{(d)} - Y - 1500 \cdot P_{x+9}^{(c)}
\]

\[
C^m = \{Pr_{10} \cdot (A-m)\} - \left[ Pr_{10} \cdot (E-m) \right]
\]

\[
= \{ X - 10000 (0.02) - Y - 1500 \left( 1 - 0.02 - 0.1 \right) \}
\]

\[
- \left[ X - 10000 (0.015) - Y - 1500 \left( 1 - 0.015 - 0.1 \right) \right]
\]

\[
= \{ X - Y - 1520 \} - \left[ X - Y - 1477.5 \right]
\]

\[
\therefore \quad C^m = -42.5
\]
4. For a fully discrete 5-year term insurance on $(x)$, you are given:

(i) The annual gross premium is $1200$.

(ii) The profit vector is $Pr = (-900, 265, 265, 265, 265, 265)$.

(iii) Mortality follows a constant force model with $\mu = -\ln(0.9)$.

(iv) The hurdle rate is $i = 0.05$.

Determine

(a) (10 points) $NPV(3)$

(b) (10 points) The profit margin for this policy.

\[ NPV(3) = \pi_0 + \pi_1 u + \pi_2 u^2 + \pi_3 u^3 + \pi_4 u^4 + \pi_5 u^5 \]

\[ = -900 + \frac{265}{1.05} + \frac{265(1.1)}{1.05^2} + \frac{265(1.1)^2}{1.05^3} \]

\[ \therefore NPV(3) = -245.869 \ldots \]

\[ PM = \frac{NPV}{APV(\text{premiums})} \]

\[ NPV = \pi_0 + \pi_1 u + \pi_2 u^2 + \pi_3 u^3 + \pi_4 u^4 + \pi_5 u^5 = 492.92 \ldots \]

\[ APV(\text{premiums}) = 1200 \cdot \bar{a}_{x:51} = 1200 \left( 1 + 2\bar{d}_x + 2^2 2\bar{d}_x + 2^3 3\bar{d}_x + 2^4 4\bar{d}_x \right) \]

\[ = 1200 \left( 1 + \frac{9}{1.05} + \frac{9^2}{1.05^2} + \frac{9^3}{1.05^3} + \frac{9^4}{1.05^4} \right) = 4513.619 \ldots \]

\[ \therefore PM = 0.01092 \ldots \]