MAP 4176 / 5178 Name:________________________

Test 12 Date: April 10, 2019

Each problem is worth 10 points. Show all work for full credit, and use correct notation.

1. For a fully continuous whole life insurance of 1,000 issued to (30), you are given:
   (i) The level annual premium rate is determine using the equivalence principle.
   (ii) $\bar{A}_{30} = 0.10$
   (ii) $\bar{A}_{50} = 0.20$
   (iii) ${}^{2}\bar{A}_{50} = 0.06$

   Determine $Var({}_{20}L)$.

   $Var\left({}_{20}L\right) = 1000^2 \cdot \frac{{}^{2}\bar{A}_{50} - (\bar{A}_{50})^2}{(1-\bar{A}_{30})^2}$

   $\therefore Var\left({}_{20}L\right) = 24{,}690$

2. For a fully discrete 20-year endowment insurance of 5000 issued to (20), use the SULT actuarial assumptions to determine the net premium reserve at time $t = 10$.

   $${}_{10}V = 5000\left(1 - \frac{\ddot{a}_{30:\overline{10|}}}{\ddot{a}_{20:\overline{20|}}}\right) = 1899$$

   There are several other correct ways to answer this question. The method above is the most time-ef

3. For a fully continuous whole life insurance of 1000 on (40), you are given:
   (i) $\mu = 0.04$ and $\delta = 0.06$
   (ii) the annual gross premium (rate), payable continuously for a maximum of 10 years, is 72
   (iii) expenses are
   (a) an initial expense of 20
   (b) 3 per year, payable continuously for the lifetime of (40)

   Determine ${}_{20}V^{g}$, the gross premium reserve at time $t = 20$.

   Since there are no more future premiums at time $t = 20$, then

   $${}_{20}V^{g} = APV(FB + FE) = 1000\bar{A}_{60} + 3\bar{a}_{60} = 1000\frac{\mu}{\mu+\delta} + 3\frac{1}{\mu+\delta} = 430$$

4. For a fully discrete whole life insurance issued to (20), you are given:

(i) The death benefit is 100,000 in year 1; 200,000 in year 2; and X thereafter
(ii) The premiums are 250 in year 1; 600 in year 2; and Y thereafter
(iii) Using the SULT actuarial assumptions, ${}_0V = E[\,{}_0L\,] = 0$

Using the SULT actuarial assumptions, determine the time 2 reserve, ${}_2V = E[\,{}_2L\,]$.

By (iii), we can use a retrospective calculation.

$$ {}_2V = AAV(PP) - AAV(PB) = \frac{250 + 600vp_{20}}{v^2 \cdot {}_2p_{20}} - \frac{100000vq_{20} + 200000v^2 \cdot {}_{1|}q_{20}}{v^2 \cdot {}_2p_{20}} $$

$$ \therefore \; {}_2V = 829 $$

5. For a fully discrete 3-year endowment insurance of 1000 on (x), you are given:

(i) Expenses, payable at the beginning of the year, are:

Year(s)	Percent of Premium	Per Policy
1	20%	15
2 and 3	8%	5

(ii) The expense reserve at the end of year 2 is -23.64.
(iii) The gross annual premium calculated using the equivalence principle is $G = 368.05$.
(iv) $G = 1000P_{x:\overline{3}|} + P^e$, where P^e is the expense loading.

Calculate $P_{x:\overline{3}|}$.

At time $t = 2$, the future expenses are $0.08(368.05) + 5 = 34.444$, payable at time 2, and the future expense premium is P^e, payable at time 2.

So, by (ii), we have $-23.64 = {}_2V^e = 34.444 - P^e$, and so $P^e = 58.084$.

$$ \therefore G = 368.05 = 1000P_{x:\overline{3}|} + 58.086 \quad \Rightarrow \quad P_{x:\overline{3}|} = 0.309966 $$