MAP 4176 / 5178 Name:______________________________

Test 13-14 (Takehome) Due: In class on April 23, 2019 Date: April 18, 2019

Each problem is worth 10 points. Show all work for full credit, and use correct notation.

1. For a whole life insurance of 10,000 with semi-annual premiums on (80), you are given:

   (i) Gross premiums of 600 are payable every 6 months starting at age 80

   (ii) There is a percent of premium expense of 10%

   (iii) The death benefit is paid at the end of the quarter of death

   (iv) $i^{(4)} = 0.08$

   (v)

t	l_{90+t}
0	1000
0.25	900
0.50	810
0.75	720

   (vi) ${}_{10.75}V = 7500$

   Calculate ${}_{10.25}V$.

Solution:

Using $v = \frac{1}{1.02} = qdf$, we have

$$
\begin{aligned}
{}_{10.25}V &= 10000 \cdot v \cdot {}_{0.25}q_{90.25} + 10000 \cdot v^2 \cdot {}_{0.25|0.25}q_{90.25} + 0.1(600) \cdot v \cdot {}_{0.25}p_{90.25} \\
&\quad - 600 \cdot v \cdot {}_{0.25}p_{90.25} + {}_{10.75}V \cdot v^2 \cdot {}_{0.50}p_{90.25}
\end{aligned}
$$

$$
\begin{aligned}
&= 10000 \cdot \frac{1}{1.02} \cdot \frac{900 - 810}{900} + 10000 \cdot \left(\frac{1}{1.02}\right)^2 \cdot \frac{810 - 720}{900} + 0.1(600) \cdot \frac{1}{1.02} \cdot \frac{810}{900} \\
&\quad - 600 \cdot \frac{1}{1.02} \cdot \frac{810}{900} + 7500 \cdot \left(\frac{1}{1.02}\right)^2 \cdot \frac{720}{900}
\end{aligned}
$$

$$\therefore {}_{10.25}V = 7232$$

2. Andy and Cathy are two 30-year olds. They each purchase a 35-year deferred whole life annuity due with annual payments of 10,000 by paying level annual net premiums at the beginning of each year during the deferral period.

   Cathy's annuity contains a return of premium feature whereby during the deferral period, a death benefit equal to the return of premiums with interest is paid at the end of the year of death. Andy' annuity contains no such feature.

   Using SULT actuarial assumptions, determine the value of $C - A$, where C denotes Cathy's premium and A denotes Andy's premium.

   Solution:
   For Cathy, ${}_{35}V = C \cdot \ddot{s}_{\overline{35|}} = 10000 \cdot \ddot{a}_{65}$, and using the SULT, we have $C = 1429$.

   For Andy, $A \cdot \ddot{a}_{30:\overline{35|}} = 10000 \cdot {}_{35|}\ddot{a}_{30} = 10000 \cdot {}_{35}E_{30} \cdot \ddot{a}_{65}$, and using the SULT, we have $A = 1366$.

$$\therefore C - A = 63$$

3. For a fully continuous 5-year endowment insurance issued to (40), you are given:

   (i) The death benefit at time t is $S(t) = 100t$

   (ii) The pure endowment is 500

   (iii) $\mu_{x+t} = 0.001 \cdot (1.1)^{x+t}$ and $\delta_t = 0.01t$

   (iv) The annual gross premium rate at time t is $\pi_t = 2 + t$

   (v) Non-settlement expenses are paid continuously at a rate of $e_t = 1 + 0.5t$

   (vi) The settlement expense, paid upon death at time t, is $10t$

   (vii) There is no settlement expense for the pure endowment

   (viii) All reserve statements refer to gross premium reserve

   Use Euler's Backward Equation with step size 0.5 to approximate ${}_4V$.

Solution:
Euler's Method (EM) states that ${}_{t+h}V = {}_tV + h \cdot {}_t\dot{V}$, and we'll be using $h = -0.5$ since we're using a backward equation with step size 0.5. Since the pure endowment is 500, then ${}_5V = 500$ is our starting value.

Note that ${}_t\dot{V} = {}_tV \cdot \delta_t + \pi_t - e_t - (S_t + E_t - {}_tV) \cdot \mu_{40+t}$

Then ${}_5\dot{V} = 500 \cdot 0.01(5) + 7 - 3.5 - (500 + 50 - 500\,) \cdot 0.001(1.1)^{45} = 28.855 \cdots$

So the first step of EM with $t = 5$ and $h = -0.5$, gives

$$ {}_{4.5}V = {}_5V - 0.5 \cdot {}_5\dot{V} = 500 - 0.5 \cdot (28.855 \cdots) = 487.572 \cdots $$

Then ${}_{4.5}\dot{V} = (485.882 \cdots) \cdot 0.01(4.5) + 6.5 - 3.25 -$
$(450 + 45 - 485.882 \cdots) \cdot 0.001(1.1)^{44.5} = 24.674 \cdots$

So the next iteration of EM with $t = 4.5$ and $h = -0.5$, gives

$$ {}_4V = {}_{4.5}V - 0.5 \cdot {}_{4.5}\dot{V} = (487.572 \cdots) - 0.5 \cdot (24.674 \cdots) = 475.235 \cdots $$

4. A life insurance company issues 5000 fully discrete whole life insurance policies of 10,000 to independent lives age 50. You are given:

(i) The annual gross premium is 250 per policy

(ii) At the end of the third policy year, there are 4800 policies in force

(iii) During the fourth policy year:

There was a per policy expense of 25 paid at the BOY
There were 50 deaths.
The earned interest rate was 6%.

(iv) The reserve values are ${}_3V = 500$ and ${}_4V = 670$

Calculate the total profit for the fourth policy year for all policies in force at the beginning of the year.

Solution:
Since $Pr_4 = \left({}_3V + \pi - e\right)(1+i) - S \cdot q_{53} - {}_4V \cdot p_{53}$, then

$$Pr_4 = 725(1.06) - 10000 \cdot \frac{50}{4800} - 670 \cdot \frac{4750}{4800} = 1.3125$$

The above value is the profit at time $t = 4$ per policy in force at time $t = 3$. Since there were 4800 policies in force at time $t = 3$, the total profit for the fourth policy year for all policies in force at the beginning of the year is

$$4800 \cdot (1.3125) = 6300$$

5. A life insurance company sells a portfolio of 1000 fully discrete whole life insurance policies of 500, on lives age 45. You are given:

   (i) There are no expenses.

   (ii) The annual gross premium is 9 per policy

   (iii) At the end of the third policy year, there are 980 policies in force, and the reserve per policy is 20.

   (iv) During the fourth policy year, there were 10 deaths, and at the end of the fourth policy year, the reserve per policy is 30.

   Determine the interest rate earned during the fourth policy year, assuming there was no gain or loss during the year.

Solution:

Since there are no expenses, $Pr_4 = \left({}_3V + \pi \right)(1+i) - S \cdot q_{48} - {}_4V \cdot p_{48}$.

$\therefore Pr_4 = 29(1+i) - 500 \cdot \frac{10}{980} - 30 \cdot \frac{970}{980} = 29(1+i) - 34.795\cdots$

Since there was no gain or loss during the year, the profit equals 0.

$$\therefore i = \frac{34.795\cdots}{29} - 1 = 0.20$$

6. A life insurance company issues fully discrete 20-year term insurance policies of 1000 to 10,000 independent 45 year olds. You are given:

   (i) Expected mortality follows the SULT.

   (ii) During the first two years, there were a total of 10 actual deaths.

   (iii) During the third year, there were 6 deaths.

   (iv) The gross reserve per policy at the end of year 3 is ${}_3V = 4$

   Calculate the company's total gain due to mortality for the third year, for all policies in force at the beginning of the third year.

Solution:
The gain for the third year per policy in force at the beginning of the year is

$G^m = \Pr(A - m, E - i, e) - \Pr(E - m, i, e)$ where

$\Pr = Pr_3 = \left({}_2V + \pi - e\right)(1 + i) - (S + E) \cdot q_{47} - {}_3V \cdot p_{47}$

Since there's no mention of expenses, assume there are none. Also, since we're calculating gain due to mortality, only the terms with q's and p's will change.

$\therefore G^m = \left\{-S \cdot q_{47}^A - {}_3V \cdot p_{47}^A\right\} - \left\{-S \cdot q_{47}^E - {}_3V \cdot p_{47}^E\right\}$
$= S \cdot (q_{47}^E - q_{47}^A) + {}_3V \cdot (p_{47}^E - p_{47}^A) = 1000 \cdot (q_{47}^E - q_{47}^A) + 4 \cdot (p_{47}^E - p_{47}^A)$

From the SULT, $q_{47}^E = 0.000916$, and so $p_{47}^E = 0.999084$

From the information in the problem, $q_{47}^A = \frac{6}{9990}$, and so $p_{47}^A = \frac{9984}{9990}$

$\therefore G^m = 0.314\cdots$ is the gain per policy, and since there are 9990 policies in force at the beginning of the year, the company's total gain due to mortality for the third year, for all policies in force at the beginning of the third year is

$$9990 \cdot (0.314\cdots) = 3138$$

7. For a fully discrete 3-year term insurance policy issued to (55), you are given:

(i) $q_{55+k} = 0.01 + 0.005k$, for $k = 0,1$

(ii) Pre-contract expenses are 100.

(iii) $Pr_1 = -16 \qquad Pr_2 = -12 \qquad Pr_3 = 168$

(iv) The hurdle rate is 10%.

Calculate the net present value of this policy.

Solution:

$NPV = \pi_0 + \pi_1 \cdot v + \pi_2 \cdot v^2 + \pi_3 \cdot v^3$ where

$\pi_0 = Pr_0 = -100$
$\pi_1 = Pr_1 = -16$
$\pi_2 = Pr_2 \cdot p_{55} = -12 \cdot 0.99 = -11.88$
$\pi_3 = Pr_3 \cdot {}_2p_{55} = 168 \cdot 0.99 \cdot 0.985 = 163.8252$
and $v = \frac{1}{1.10}$

$$\therefore NPV = -1.28$$

8. For a fully discrete 2-year term life insurance on (50), you are given:

   (i) The annual gross premium is 250.

   (ii) The hurdle rate is 10%.

   (iii) The profit vector is $\overrightarrow{Pr} = \langle -165, 100, 125 \rangle$.

   (iv) The profit margin for this insurance is 6%.

   Calculate the probability that (50) will survive one year.

Solution:
The profit margin is defined as $PM = \frac{NPV}{APV(Premiums)}$ where both numerator and denominator are calculated using the hurdle rate.

$\therefore\ 0.06 = \frac{-165+100\cdot v+125\cdot p_{50}\cdot v^2}{250+250\cdot v\cdot p_{50}}$

$$v = \frac{1}{1.10} \quad \Rightarrow \quad p_{50} = 0.99355$$

9. For a 5-year insurance policy on (65), you are given:

(i) The profit signature is $\vec{\Pi} = \langle -450, 250, 175, 75, 100, 150 \rangle$

(ii) The risk discount rate is 10%.

Calculate the discounted payback period for this policy.

Solution:

Note: $v = \frac{1}{1.10}$

$NPV(0) = \pi_0 = -450$

$NPV(1) = NPV(0) + \pi_1 \cdot v = -450 + 250 \cdot v = -222.72 \cdots$

$NPV(2) = NPV(1) + \pi_2 \cdot v^2 = (-222.72 \cdots) + 175 \cdot v^2 = -78.09 \cdots$

$NPV(3) = NPV(2) + \pi_3 \cdot v^3 = (-78.09 \cdots) + 75 \cdot v^3 = -21.75 \cdots$

$NPV(4) = NPV(3) + \pi_4 \cdot v^4 = (-21.75 \cdots) + 100 \cdot v^4 = 46.55 \cdots$

$$\therefore DPP = 4$$

10. For a 3-year term insurance on (x), you are given:

(i) the profit vector is $\overrightarrow{Pr} = \langle -500, 375, 100, 100 \rangle$

(ii) mortality follows a constant force model

(iii) the internal rate of return for this product is 6%

Determine q_x.

Solution:
The internal rate of return is the interest rate for which the net present value is 0.

For this product, we have

$$NPV = -500 + 375 \cdot v + 100 \cdot p_x \cdot v^2 + 100 \cdot {}_2p_x \cdot v^3$$

Since mortality follows a constant force model, ${}_kp_x = p^k$. Using $i = 0.06$, we have

$$0 = -500 + 375 \cdot \frac{1}{1.06} + 100 \cdot p \cdot \left(\frac{1}{1.06}\right)^2 + 100 \cdot p^2 \cdot \left(\frac{1}{1.06}\right)^3$$

Solving this quadratic equation in p, we get

$$p = 0.892 \cdots \quad \Rightarrow \quad q = q_x = 0.107 \cdots$$