MAP 4176 / 5178 Name:______________________

Test 12 Date: December 2, 2015

Show all work for full credit, use correct notation., and clearly mark your answer.

1. Using ILT assumptions determine
(a) the single benefit premium for a whole-life insurance of 100,000 issued to (40) with benefit payable at the end of the year of death.

$$SBP = 100000 A_{40} \overset{ILT}{=} 16132$$

(b) the variance of the present value random variable for the insurance in part (a)

$$Var(Z) = 100000^2 \left[{}^2A_{40} - (A_{40})^2 \right] \overset{ILT}{=} 100000^2 \left[.04863 - (.16132)^2 \right]$$
$$= 226\,058\,576$$

2. Using ILT assumptions determine
(a) the single net premium for a 20-year pure endowment of 5000 issued to (30).

$$SNP = 5000\, {}_{20}E_{30} \overset{ILT}{=} 5000(.29374) = 1468.70$$

(b) the variance of the present value random variable for the insurance in part (a)

$$Var(Z) = 5000^2 \left[{}^2_{20}E_{30} - ({}_{20}E_{30})^2 \right] = 5000^2 \left[v^{20} \cdot {}_{20}E_{30} - ({}_{20}E_{30})^2 \right]$$
$$\overset{ILT}{=} 5000^2 \left[(1.06)^{-20} (.29374) - (.29374)^2 \right] = 132658$$

3. Using ILT assumptions, determine
(a) the actuarial present value for a 10-year deferred whole-life insurance of 1,000 issued to (35) with benefit payable at the end of the year of death.

$$APV = 1000\, {}_{10|}A_{35} = 1000\, A_{45} \cdot {}_{10}E_{35} \overset{ILT}{=} (201.20)(.54318) = 109.29$$

(b) the variance of the present value random variable for the insurance in part (a)

$$Var(Z) = 1000^2 \left[{}^2A_{45} \cdot {}^2_{10}E_{35} - (A_{45} \cdot {}_{10}E_{35})^2 \right]$$
$$\overset{ILT}{=} 1000^2 \left[(.06802)((1.06)^{-10}(.54318)) - ((.2012)(.54318))^2 \right]$$
$$= 8687$$

4. Using DML(100) mortality and $i = .06$, determine

(a) the EPV for a discrete 10-year term insurance of 10,000 issued to (40).

$$EPV = 10000\, A^{1}_{40:\overline{10}|} = 10000\left(v q_{40} + v^2\, {}_{1|}q_{40} + \cdots + v^{10}\, {}_{9|}q_{40}\right)$$

$$DML(100) \Rightarrow q_{40} = {}_{1|}q_{40} = \cdots = {}_{9|}q_{40} = \frac{1}{60}$$

$$\therefore\ EPV = 10000\, A^{1}_{40:\overline{10}|} = 10000 \cdot \frac{1}{60}\left(v + v^2 + \cdots + v^{10}\right) = 10000 \cdot \frac{1}{60} \cdot a_{\overline{10}|.06}$$

$$= 1226.68$$

(b) the variance of the present value random variable for the insurance in part (a)

$$Var(Z) = 10000^2\left[{}^{2}A^{1}_{40:\overline{10}|} - \left(A^{1}_{40:\overline{10}|}\right)^2\right]$$

$$A^{1}_{40:\overline{10}|} = \frac{1}{60} \cdot a_{\overline{10}|.06} \Rightarrow {}^{2}A^{1}_{40:\overline{10}|} = \frac{1}{60} \cdot a_{\overline{10}|.1236}$$

$$\therefore\ Var(Z) = 10000^2\left[\frac{1}{60} a_{\overline{10}|.1236} - \left(\frac{1}{60} a_{\overline{10}|.06}\right)^2\right] = 7775125$$

5. Using constant force assumptions with $\mu = .02$ and $i = .05$, determine

(a) the EPV for a 20-year deferred whole life insurance of 500 issued to (x) with benefit payable at the end of the year of death.

$$EPV = 500 \cdot {}_{20|}A_x = 500\, A_{x+20} \cdot {}_{20}E_x$$

$$A_y \overset{CF}{=} \frac{q}{q+i} \quad (\text{for any } y) \qquad {}_{20}E_x = v^{20} \cdot {}_{20}p_x = (1.05)^{-20} \cdot e^{-20(.02)}$$

$$q = 1 - p = 1 - e^{-.02} \qquad \therefore\ EPV = 500\, \frac{1-e^{-.02}}{1-e^{-.02}+.05}\, (1.05)^{-20} \cdot e^{-.4} = 35.83$$

(b) the variance of the present value random variable for the insurance in part (a)

$$Var(Z) = 500^2\left[{}^{2}_{20|}A_x - \left({}_{20|}A_x\right)^2\right]$$

$$= 500^2\left[{}^{2}A_{x+20} \cdot {}^{2}_{20}E_x - \left(A_{x+20} \cdot {}_{20}E_x\right)^2\right]$$

$$A_{x+20} = \frac{q}{q+i} \Rightarrow {}^{2}A_{x+20} = \frac{q}{q+2i+i^2} \qquad {}^{2}_{20}E_x = v^{20} \cdot {}_{20}E_x$$

$$\therefore\ Var(Z) = 500^2\left[\frac{1-e^{-.02}}{1-e^{-.02}+.1+.0025}\,(1.05)^{-20}(1.05)^{-20} e^{-.4} - \left(\frac{1-e^{-.02}}{1-e^{-.02}+.05}\,(1.05)^{-20} \cdot e^{-.4}\right)^2\right]$$

$$= 2570$$