MAP 4176 / 5178 Name:________________________

Test 1 Date: January 17, 2015

Show all work for full credit, use correct notation, and clearly mark your answer.

1. (10 points) Using ILT assumptions determine the variance of the present value random variable for a whole-life insurance of 50,000 issued to (35) with benefit payable at the end of the year of death.

$$Z = 50000\, Z_{35}$$

$$Var(Z) = 50000^2\left[{}^2A_{35} - (A_{35})^2\right]$$

$$= 50000^2\left[.03488 - (.12872)^2\right] = 45{,}777{,}904$$

2. Using ILT assumptions determine

(a) (10 points) the single net premium for a discrete 20-year endowment insurance of 10,000 issued to (40).

$$SNP = 10000\, A_{40:\overline{20|}} = 10000 \cdot \left(A^{1}_{40:\overline{20|}} + A_{40:\overset{\,1}{\overline{20|}}}\right)$$

$$A^{1}_{40:\overline{20|}} = A_{40} - {}_{20}E_{40} \cdot A_{60} = .16132 - (.27414)(.36913) = .0601\cdots$$

$$A_{40:\overset{\,1}{\overline{20|}}} = {}_{20}E_{40} = .27414$$

$$\therefore\ SNP = 10000 \cdot (.0601\cdots + .27414) = 3342.67$$

(b) (10 points) the variance of the present value random variable for the insurance in part (a).

$$Var(Z) = 10000^2\left[{}^2A_{40:\overline{20|}} - (A_{40:\overline{20|}})^2\right]$$

$$A_{40:\overline{20|}} \overset{\text{see (a)}}{=\!=} A_{40} - {}_{20}E_{40} \cdot A_{60} + {}_{20}E_{40} \overset{\text{see}}{\underset{(a)}{=}} .3342\cdots$$

$$^2A_{40:\overline{20|}} = {}^2A_{40} - {}^2_{20}E_{40} \cdot {}^2A_{60} + {}^2_{20}E_{40} \qquad {}^2_{20}E_{40} = (1.06)^{-20} \cdot {}_{20}E_{40}$$

$$= .04863 - (1.06)^{-20} \cdot (.27414)(.17741) + (1.06)^{-20} \cdot (.27414)$$

$$= .1189\cdots$$

$$\therefore\ Var(Z) = 10000^2\left(.1189\cdots - (.3342)^2\right) \doteq 720925$$

3. Using ILT assumptions, determine
(a) (10 points) the actuarial present value for a 25-year deferred whole-life insurance of 1,000 issued to (25) with benefit payable at the end of the year of death.

$$Z = 1000\, {}_{25|}Z_{25} \Rightarrow APV = 1000\, {}_{25|}A_{25} = 1000\, {}_{25}E_{25} \cdot A_{50}$$

$${}_{25}E_{25} = {}_{20}E_{25} \cdot {}_{5}E_{45} \overset{ILT}{=} (.29873)(.72988) = .218\cdots$$

(Note: We could also use

$${}_{25}E_{25} = {}_{5}E_{25} \cdot {}_{20}E_{30} \quad \text{or} \quad {}_{25}E_{25} = v^{25} \cdot {}_{25}p_{25} = (1.06)^{-25} \cdot \frac{l_{50}}{l_{25}} \,)$$

$$\therefore APV = 1000(.218\cdots)(.24905) = 54.30$$

(b) (10 points) the variance of the present value random variable for the insurance in part (a).

$$Var(Z) = 1000^2 \cdot Var({}_{25|}Z_{25}) = 1000^2 \left[{}^{2}_{25|}A_{25} - ({}_{25|}A_{25})^2 \right]$$

From above, ${}_{25|}A_{25} = {}_{25}E_{25} \cdot A_{50} = (.218\cdots)(.24905) = .0543\cdots$

Then $${}_{25|}{}^{2}A_{25} = {}^{2}_{25}E_{25} \cdot {}^{2}A_{50} = v^{25} \cdot {}_{25}E_{25} \cdot {}^{2}A_{50}$$

$$= (1.06)^{-25} \cdot (.218\cdots) \cdot (.09476) = .0048\cdots$$

$$\therefore Var(Z) = 1000^2 \left[(.0048\cdots) - (.0543\cdots)^2 \right] = 1865$$