MAP 4176 / 5178 Name:____________________

Test 1 Date: January 24, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

For numbers 1 and 2, use ILT actuarial assumptions to determine the actuarial present value of the annuity.

1. a whole life annuity immediate of 10,000 issued to (30)

$$APV = 10000\, a_{30} = 10000(\ddot{a}_{30} - 1) \overset{ILT}{=} 148{,}561$$

2. a 15-year certain-and-life annuity due issued to (25) with annual payments of 1,000

$$APV = 1000\, \ddot{a}_{\overline{25:\overline{15|}}}$$

$$\ddot{a}_{\overline{25:\overline{15|}}} = \ddot{a}_{\overline{15|}} + {}_{15}E_{25} \cdot \ddot{a}_{40}$$

$$\overset{ILT}{=} 16.3146\ldots$$

$$\therefore APV = 16315$$

$${}_{15}E_{25} = {}_{10}E_{30} \cdot {}_{5}E_{25}$$

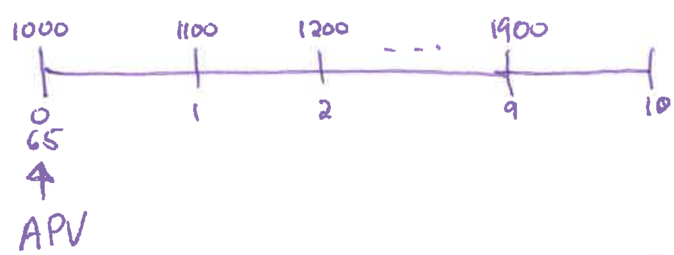
OR

$${}_{15}E_{25} = {}_{5}E_{35} \cdot {}_{10}E_{25}$$

OR

$${}_{15}E_{25} = v^{15} \cdot {}_{15}p_{25}$$

3. A 10-year temporary life annuity due issued to (65) has a payment of 1000 for the first year. For subsequent years, the payment is 100 more than the previous year's payment. Write an expression using actuarial notation for the APV of this annuity.



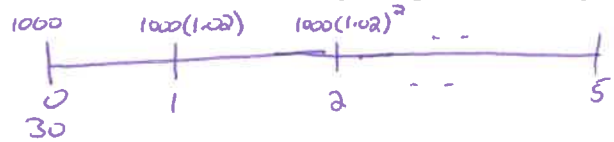
There are several correct expressions. Two are:

$$APV = 1000\, \ddot{a}_{65:\overline{10|}} + 100\,(Ia)_{65:\overline{9|}}$$

OR

$$APV = 900\, \ddot{a}_{65:\overline{10|}} + 100\,(I\ddot{a})_{65:\overline{10|}}$$

4. A 5-year temporary life annuity due with annual payments issued to (30) has a first payment of 1000. Subsequent payments are 2% larger than their previous payments. Determine the APV of this annuity using ILT mortality and $i = 8.12\%$.



$v_i = \frac{1}{1.0812} \Rightarrow 1.02\,v_i = \frac{1}{1.06} = v_{.06}$

$APV \overset{VEP}{=} 1000 + 1000(1.02)\,v_i\,p_{30} + 1000(1.02)^2 \cdot v_i^2 \cdot {}_2p_{30} + \cdots$

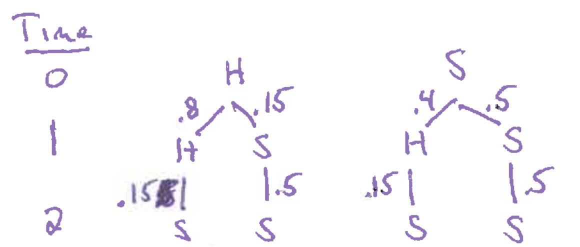
$\therefore APV = 1000\left(1 + v_{.06} \cdot p_{30} + v_{.06}^2 \cdot {}_2p_{30} + v_{.06}^3 \cdot {}_3p_{30} + v_{.06}^4 \cdot {}_4p_{30}\right)$

$= 1000 \cdot \ddot{a}_{30:\overline{5|}} = 1000\left(\ddot{a}_{30} - {}_5E_{30} \cdot \ddot{a}_{35}\right) \overset{ILT}{=} 4452$

5. In a homogeneous Markov model with 3 states: Healthy (H), Sick (S), and Dead (D), you are given:

(i) the annual transition probabilities are

	H	S	D
H	.80	.15	.05
S	.40	.50	.10
D	0	0	1



$p_x^{HS} = .15 \qquad p_x^{SS} = .5$

$.8(.15) + .15(.5)$

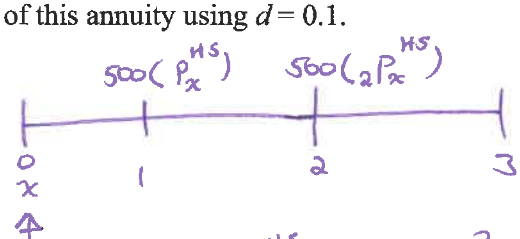
${}_2p_x^{HS} = .195 \qquad {}_2p_x^{SS} = .31$

(ii) in a population of x-year olds, 80% are healthy and 20% are sick

A 3-year temporary life annuity due with annual payments issued to (x) pays 500 if (x) is sick. For a randomly selected person selected from the population of x-year olds, determine the APV of this annuity using $d = 0.1$.

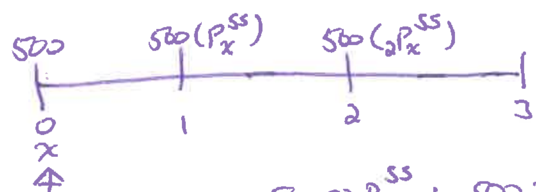
$v = 1 - .1 = .9$

For H x-year olds:



$APV = 500\,v\,p_x^{HS} + 500\,v^2 \cdot {}_2p_x^{HS} = 146.475$

For S x-year olds:



$APV = 500 + 500\,v\,p_x^{SS} + 500\,v^2 \cdot {}_2p_x^{SS} = 850.55$

$\therefore$ For a randomly chosen person, $APV = .8(146.475) + .2(850.55) = 287.29$