MAP 4176 / 5178
Test 2

Name: KEY

Date: January 27, 2016

Show all work for full credit, use correct notation., and clearly mark your answer.

1. Using ILT assumptions determine
   (a) the expected present value for a whole-life insurance of 5,000 issued to independent lives, both age 40, with benefit payable at the end of the year of the second death.

$$EPV = E[Z] = 5000\,A_{\overline{40:40}} = 5000\left(A_{40} + A_{40} - A_{40:40}\right)$$

$$\overset{ILT}{=} 5000(.16132 + .16132 - .22999) = 463.25$$

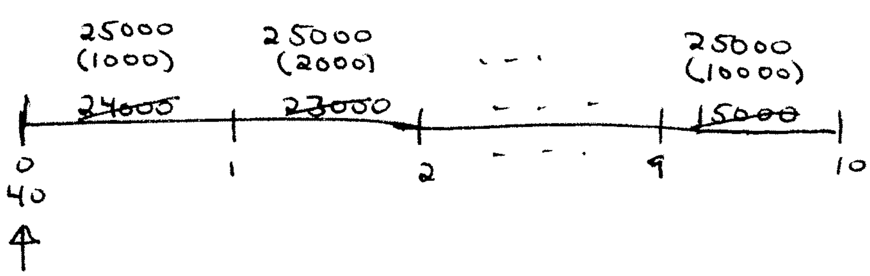
   (b) the variance of the present value random variable for the insurance in part (a)

$$E[Z^2] = 5000^2\left({}^2A_{40} + {}^2A_{40} - {}^2A_{40:40}\right)$$

$$\overset{ILT}{=} 5000^2(.04863 + .04863 - .08489) = 309250$$

$$\therefore Var(Z) = 309250 - (463.25)^2 \doteq 94650$$

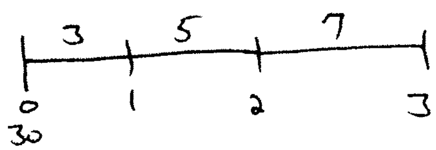
2. Determine the actuarial present value of a 10-year term insurance issued to (40) with death benefit payable at the end of the year of death. The death benefit is $25000 - 1000n$ if death occurs during year n, for $n = 1,2,\ldots,10$.
   (You are given $A^{\;1}_{40:\overline{10|}} = 0.17094$ and $(IA)^{\;1}_{40:\overline{10|}} = 0.96728$.)



$$APV = 25000 \cdot A^{\;1}_{40:\overline{10|}} - 1000(IA)^{\;1}_{40:\overline{10|}}$$

$$= 25000(.17094) - 1000(.96728) = 3306.22$$

3. Using $i = .05$ and ILT mortality, determine the variance of the present value random variable for a 3-year discrete term insurance issued to (30) with death benefit equal to 3 in the first year, 5 in the second year, and 7 in the third year.



PVRV Z	Pr
$3/1.05$	$q_{30} = .00153$
$5/1.05^2$	$_{1\|}q_{30} = .99847(.00161)$
$7/1.05^3$	$_{2\|}q_{30} = .99847(.99839)(.0017)$
0	$_{3}p_{30}$

$$E[Z] = \frac{3}{1.05}(.00153) + \frac{5}{1.05^2}[.99847(.00161)] + \frac{7}{1.05^3}[.99847(.99839)(.0017)] \rightarrow \boxed{1}$$

$$E[Z^2] = \left(\frac{3}{1.05}\right)^2(.00153) + \left(\frac{5}{1.05^2}\right)^2[.99847(.00161)] + \left(\frac{7}{1.05^3}\right)^2[.99847(.99839)(.0017)] \rightarrow \boxed{2}$$

$$Var(Z) = \boxed{2} - (\boxed{1})^2 = .107037605$$

4. You are given:

j	$A_x^{(\text{Dec } j)}$	$A_{x:\overline{n}\vert}^{(\text{Dec } j)}$
1	0.150	0.420
2	0.465	0.585

You are also given $_nE_x = 0.3$. A discrete whole life insurance issued to (x) pays 50 if departure occurs within n years by decrement 2, pays 100 if departure occurs after n years by decrement 1, and pays nothing otherwise. Determine the EPV of the insurance

$$EPV = 50\, A_{\overset{1}{x}:\overline{n}\vert}^{(\text{Dec}2)} + 100\, {}_{n|}A_x^{(\text{Dec }1)}$$

$$A_{\overset{1}{x}:\overline{n}\vert}^{(\text{Dec}2)} = A_{x:\overline{n}\vert}^{(\text{Dec}2)} - {}_nE_x = .585 - .3 = .285$$

$${}_{n|}A_x^{(\text{Dec}1)} = A_x^{(\text{Dec}1)} - A_{\overset{1}{x}:\overline{n}\vert}^{(\text{Dec}1)} = A_x^{(\text{Dec}1)} - \left(A_{x:\overline{n}\vert}^{(\text{Dec}1)} - {}_nE_x\right)$$

$$= .15 - (.42 - .3) = .03$$

$$\therefore\ EPV = 50(.285) + 100(.03) = 17.25$$