MAP 4176 / 5178 Name:______________________

Test 2 Date: January 31, 2018

Show all work for full credit, use correct notation, and clearly mark your answer. Each problem is worth 10 points.

1. Using ILT assumptions and the 2-term Woolhouse approximation, determine the actuarial present value of a 10-year temporary life annuity issued to (30) that pays 1000 at the beginning of each month.

$$APV = 12000\,\ddot{a}^{(12)}_{30:\overline{10|}} = 12000\left(\ddot{a}^{(12)}_{30} - {}_{10}E_{30}\cdot\ddot{a}^{(12)}_{40}\right)$$

$${}_{10}E_{30} \overset{ILT}{=} .54733 \qquad \ddot{a}^{(12)}_{30} \overset{2WH}{=} \ddot{a}_{30} - \tfrac{11}{24} \overset{ILT}{=} 15.397\ldots$$

$$\ddot{a}^{(12)}_{40} \overset{2WH}{=} \ddot{a}_{40} - \tfrac{11}{24} \overset{ILT}{=} 14.358\ldots$$

$$\therefore APV = 90{,}469$$

2. Using ILT assumptions and the UDD assumption, determine the APV of a whole life annuity immediate with monthly payments of 500 issued to (35).

$$APV = 6000\,a^{(12)}_{35}$$

$$a^{(12)}_{35} = \ddot{a}^{(12)}_{35} - \tfrac{1}{12}$$

$$\ddot{a}^{(12)}_{35} \overset{UDD}{=} \alpha(12)\,\ddot{a}_{35} - \beta(12) \overset{ILT}{=} 14.928\ldots$$

$$\therefore APV = 89{,}073$$

3. You are given:

(i) $\mu_x = 0.02$

(ii) $\delta = 0.04$

(iii) $a_x = 8$

Using the 3-term Woolhouse approximation, determine $\bar{a}_x$.

$$\ddot{a}^{(m)}_x \overset{3WH}{=} \ddot{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2}(\mu_x+\delta) \xrightarrow{m\to\infty} \bar{a}_x = \ddot{a}_x - \tfrac{1}{2} - \tfrac{1}{12}(\mu_x+\delta)$$

$$\ddot{a}_x = 1 + a_x = 9$$

$$\therefore \bar{a}_x \overset{3WH}{=} 9 - \tfrac{1}{2} - \tfrac{1}{12}(.06) = 8.495$$

4. A whole life annuity issued to (50) pays continuously at an annual rate of $5 \cdot (1.06)^t$ at time t. Determine the APV of this annuity, given $\underbrace{{}_tp_{50} = \left(\frac{40-t}{40}\right)^3}$ and $i = 0.06$.

$5(1.06)^t$ = annual payment rate

$\Rightarrow$ mortality is GDML($\alpha=3, \omega=90$)

$0 \quad t \quad \Delta t$

$50 \qquad \infty = 40$ since (50) can live to at most age 90

90

$PMT = 5(1.06)^t \cdot \Delta t$

integrand = product $\begin{cases} PV = 5(1.06)^t \cdot \Delta t \cdot v^t \\ P_r = {}_tp_{50} \end{cases}$

$$\therefore APV = \int_0^{40} \underbrace{5(1.06)^t \cdot v^t}_{=1 \text{ since } i=.06} \cdot {}_tp_{50}\, dt = 5 \cdot \int_0^{40} {}_tp_{50}\, dt = 5 \cdot \mathring{e}_{50} = 5 \cdot \frac{\omega - x}{\alpha + 1}$$

$$\therefore APV = 5 \cdot \frac{40}{4} = 50$$

5. For a 4-state model, transitions can occur back and forth among states 0, 1, and 2. Once state 3 is entered, there is no transition out from it. You are given:

(i) ${}_5p_{80}^{00} = 0.539$ ${}_5p_{80}^{01} = 0.173$ ${}_5p_{80}^{02} = 0.070$ ${}_5p_{80}^{03} = 0.218$

(ii) Using $i = 5\%$: $\bar{a}_{85}^{02} = 0.3403$ $\bar{a}_{85}^{12} = 1.0883$ $\bar{a}_{85}^{22} = 3.2367$

(iii) $i = 5\%$

Determine the actuarial present value of a 5-year deferred whole life annuity, issued to an 80-year old in state 0, that pays continuously at a rate of 10,000 per year while in state 2.

$$APV = 10000 \cdot {}_{5|}\bar{a}_{80}^{02}$$

At age 85, we have

$APV_{85} = 10000\, \bar{a}_{85}^{02}$ if (80) is in state 0 at age 85

$\therefore$ discount actuarially back to age 80 using ${}_5E_{80}^{00} = v^5 \cdot {}_5p_{80}^{00}$

$APV_{85} = 10000\, \bar{a}_{85}^{12}$ if (80) is in state 1 at age 85

$\therefore$ discount actuarially back to age 80 using ${}_5E_{80}^{01} = v^5 \cdot {}_5p_{80}^{01}$

$APV_{85} = 10000\, \bar{a}_{85}^{22}$ if (80) is in state 2 at age 85

$\therefore$ discount actuarially back to age 80 using ${}_5E_{80}^{02} = v^5 \cdot {}_5p_{80}^{02}$

$$\therefore APV = 10000 \cdot \underbrace{\left(v^5 \cdot {}_5p_{80}^{00} \cdot \bar{a}_{85}^{02} + v^5 \cdot {}_5p_{80}^{01} \cdot \bar{a}_{85}^{12} + v^5 \cdot {}_5p_{80}^{02} \cdot \bar{a}_{85}^{22}\right)}_{= {}_{5|}\bar{a}_{80}^{02}} = 4688$$