Show all work for full credit, use correct notation, and clearly mark your answer. Each problem is worth 10 points.

1. Using ILT assumptions and the 2-term Woolhouse approximation, determine the actuarial present value of a 10-year temporary life annuity issued to (30) that pays 1000 at the beginning of each month.

\[
\text{APV} = 12000 \ddot{a}_{30}^{12} = 12000 \left( \ddot{a}_{30}^{12} - 10E_{30} \ddot{a}_{40}^{12} \right) \\
10E_{30} \xrightarrow{\text{ILT}} 0.54733 \\
\ddot{a}_{30}^{12} \xrightarrow{\text{wh}} \ddot{a}_{30}^{12} - \frac{11}{24} \xrightarrow{\text{ILT}} 15.397 \ldots \\
\ddot{a}_{40}^{12} \xrightarrow{\text{wh}} \ddot{a}_{40}^{12} - \frac{11}{24} \xrightarrow{\text{ILT}} 14.358 \ldots \\
\implies \text{APV} = 90,469
\]

2. Using ILT assumptions and the UDD assumption, determine the APV of a whole life annuity immediate with monthly payments of 500 issued to (35).

\[
\text{APV} = 6000 \ddot{a}_{35}^{12} \\
\ddot{a}_{35}^{12} = \ddot{a}_{35}^{12} - \frac{1}{12} \\
\ddot{a}_{35}^{12} \xrightarrow{\text{udd}} \alpha(12) \ddot{a}_{35}^{12} - \beta(12) \xrightarrow{\text{ILT}} 14.928 \ldots \\
\implies \text{APV} = 89,073
\]

3. You are given:

(i) \( \mu_x = 0.02 \)
(ii) \( \delta = 0.04 \)
(iii) \( a_x = 8 \)

Using the 3-term Woolhouse approximation, determine \( \bar{a}_x \).

\[
\ddot{a}_{x}^{(m)} \xrightarrow{\text{wh}} \ddot{a}_x^{(m)} - \frac{m-1}{2m} - \frac{m-1}{12A_x^2} (\mu_x + \delta) \xrightarrow{\text{wh}} \ddot{a}_x = \ddot{a}_x - \frac{1}{2} - \frac{1}{12} (\mu_x + \delta) \\
\ddot{a}_x = 1 + a_x = 9 \\
\implies \bar{a}_x \xrightarrow{\text{wh}} 9 - \frac{1}{2} - \frac{1}{12} (0.06) = 8.495
\]
4. A whole life annuity issued to (50) pays continuously at an annual rate of $5 \cdot (1.06)^t$
at time $t$. Determine the APV of this annuity, given $p_{50} = \left(\frac{40-t}{40}\right)^3$ and $i = 0.06$.

\[
\text{APV} = \int_0^{40} e^{-0.06t} \cdot (1.06)^t \cdot t \cdot p_{50} \, dt = 6 \int_0^{40} e^{-0.06t} \cdot p_{50} \, dt = 5 \cdot e_{50} = 5 \cdot \frac{\omega - x}{\alpha + 1}
\]

\[\therefore \text{APV} = 5 \cdot \frac{40}{4} = 50\]

5. For a 4-state model, transitions can occur back and forth among states 0, 1, and 2.
Once state 3 is entered, there is no transition out from it. You are given:

(i) $s_p^{00} = 0.539 \quad s_p^{01} = 0.173 \quad s_p^{02} = 0.070 \quad s_p^{03} = 0.218$

(ii) Using $i = 5\%$: $\bar{a}^{02}_{85} = 0.3403 \quad \bar{a}^{12}_{85} = 1.0883 \quad \bar{a}^{22}_{85} = 3.2367$

(iii) $i = 5\%$

Determine the actuarial present value of a 5-year deferred whole life annuity, issued to an 80-year old in state 0, that pays continuously at a rate of 10,000 per year while in state 2.

\[\text{APV} = 10000 \cdot \bar{a}^{02}_{80}\]

At age 85, we have

\[\text{APV}_{85} = 10000 \cdot \bar{a}^{02}_{85} \quad \text{if} \ (80) \ is \ in \ state \ 0 \ at \ age \ 85\]

\[\therefore \text{discount actuarially back to age 80 using } s_E_{80}^{00} = \bar{a}_8^{12} \cdot s_p^{00}_{80}\]

\[\text{APV}_{85} = 10000 \cdot \bar{a}^{12}_{85} \quad \text{if} \ (80) \ is \ in \ state \ 1 \ at \ age \ 85\]

\[\therefore \text{discount actuarially back to age 80 using } s_E_{80}^{01} = \bar{a}_8^{12} \cdot s_p^{01}_{80}\]

\[\text{APV}_{85} = 10000 \cdot \bar{a}^{22}_{85} \quad \text{if} \ (80) \ is \ in \ state \ 2 \ at \ age \ 85\]

\[\therefore \text{discount actuarially back to age 80 using } s_E_{80}^{02} = \bar{a}_8^{12} \cdot s_p^{02}_{80}\]

\[\therefore \text{APV} = 10000 \cdot \left(\bar{a}_8^{02} + \bar{a}_8^{12} \cdot s_p^{01}_{80} + \bar{a}_8^{22} \cdot s_p^{02}_{80}\right) = 4688.8\]

\[= s_1 \bar{a}_8^{02}\]