MAP 4176 / 5178 Name: KEY

Test 2 Date: January 23, 2019

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

For Numbers 1 through 3, use the actuarial assumptions in the L-TAM tables to determine the actuarial present value of each annuity described:

1. an annual payment 10-year temporary annuity due issued to independent male and female lives, both age 30, with payments of 10,000 while both are alive, 7,500 continuing to the male after the death of the female, and 5,000 continuing to the female after the death of the male.

$$APV = 10000\,\ddot{a}_{30:30:\overline{10|}} + 7500\left(\ddot{a}_{30:\overline{10|}} - \ddot{a}_{30:30:\overline{10|}}\right)$$
$$+ 5000\left(\ddot{a}_{30:\overline{10|}} - \ddot{a}_{30:30:\overline{10|}}\right)$$
$$= 12500\,\ddot{a}_{30:\overline{10|}} - 2500\,\ddot{a}_{30:30:\overline{10|}} = 80{,}990.25$$

2. a 10-year temporary annuity due issued to (20) with quarterly payments of 25,000. Use the UDD assumption, and note that $\alpha(4) = 1.00019$ and $\beta(4) = 0.38272$.

$$APV = 100000\,\ddot{a}^{(4)}_{20:\overline{10|}}$$
$$\ddot{a}^{(4)}_{20:\overline{10|}} = \ddot{a}^{(4)}_{20} - {}_{10}E_{20}\,\ddot{a}^{(4)}_{30}$$
$$\ddot{a}^{(4)}_{x} \overset{UDD}{=} \alpha(4)\cdot\ddot{a}_x - \beta(4)$$
$$\ddot{a}^{(4)}_{20} = 19.58747\ldots$$
$$\ddot{a}^{(4)}_{30} = 19.00436\ldots$$

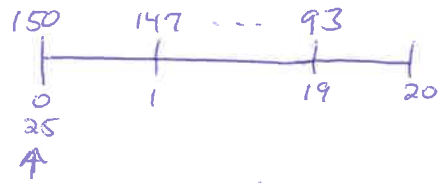
$$\therefore\ APV = 795{,}224.25$$

3. an annual payment annuity immediate of 50,000 issued to independent lives aged 30 and 40, with payments continuing until the last death

$$APV = 50000\,a_{\overline{30:40}}$$
$$a_{\overline{30:40}} = \ddot{a}_{\overline{30:40}} - 1$$
$$= \ddot{a}_{30} + \ddot{a}_{40} - \ddot{a}_{30:40} - 1$$

$$\therefore\ APV = 936{,}000$$

4. An annual payment 20-year temporary annuity due issued to (25) pays 150 for the first year, 147 for the second year, and so on, where each year's payment is 3 less than the preceding year's payment. Write an expression using actuarial notation for the APV of this annuity.



$$APV = 150\,\ddot{a}_{25:\overline{20}|} - 3(Ia)_{25:\overline{19}|}$$

(There are many other correct answers!)

5. In a non-homogeneous Markov model with 3 states: Healthy (H), Sick (S), and Dead (D), you are given: (For the matrices below, we use the standard convention that the first row/column corresponds to the Healthy state, the second row/column corresponds to the Sick state, and the third row/column corresponds to the Dead state.)

(i) the annual transition probability matrix for year 1 is:

$$Q_1 = \begin{bmatrix} 0.80 & 0.15 & 0.05 \\ 0.20 & 0.70 & 0.10 \\ 0.00 & 0.00 & 1.00 \end{bmatrix}$$

(ii) the annual transition probability matrix for year 2 is:

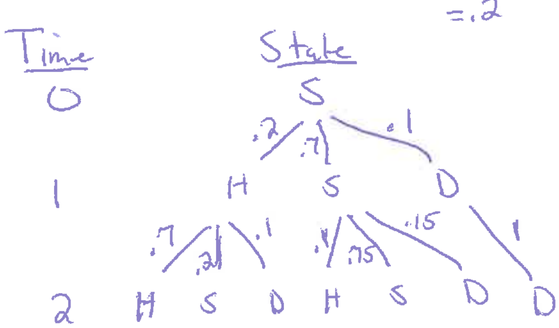
$$Q_2 = \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.10 & 0.75 & 0.15 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

(iii) (x) is currently sick

A 2-year temporary annuity immediate with annual payments issued to (x) pays 1000 if (x) is healthy and 250 if (x) is sick. Determine the actuarial present value of this annuity using an annual discount factor of 0.95. $v = .95$

$$APV = 1000\,a^{SH}_{x:\overline{2}|} + 250\,a^{SS}_{x:\overline{2}|}$$

$$a^{SH}_{x:\overline{2}|} = v\,\underbrace{p^{SH}_x}_{=.2} + v^2 \cdot {}_2p^{SH}_x \qquad a^{SS}_{x:\overline{2}|} = v\,\underbrace{p^{SS}_x}_{=.7} + v^2 \cdot {}_2p^{SS}_x$$



$$\therefore {}_2p^{SH}_x = .2(.7) + .7(.1) = 0.21$$

$${}_2p^{SS}_x = .2(.2) + .7(.75) = 0.565$$

$$\therefore APV = 1000\left(.95(.2) + .95^2(.21)\right) + 250\left(.95(.7) + .95^2(.565)\right) = 673.25$$