Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

For Numbers 1 and 2, determine the APV of the annuity described, using constant force actuarial assumptions with $\mu = 0.02$ and $\delta = 0.04$.

1. a continuous 10-year temporary annuity with annual payment rate of 500 issued to (45)

$$APV = 500 \cdot \overline{a}_{45:\overline{10}y} = 500 \cdot \left( \overline{a}_{45} - 10E_{45} \cdot \overline{a}_{55} \right)$$

Since CF, $\overline{a}_{45} = \overline{a}_{55} = \frac{1}{\mu + \delta} = \frac{1}{0.06}$ and $10E_{45} = e^{-10(\mu + \delta)} = e^{-0.6}$

$$\therefore APV = 3,760$$

2. a continuous 20-year certain-and-life annuity paying 3000 per year issued to (60)

$$APV = 3000 \cdot \overline{a}_{60:\overline{20}y} = 3000 \cdot \left( \overline{a}_{20} + 20E_{60} \cdot \overline{a}_{80} \right)$$

$$\overline{a}_{20} = \frac{1 - v^{20}}{\delta} = \frac{1 - e^{-20(0.04)}}{0.04}, \quad 20E_{60} = e^{-20(0.06)}, \quad \text{and} \quad \overline{a}_{80} = \frac{1}{0.06}$$

$$\therefore APV = 56,360$$

3. Given independent lives $(x)$ and $(y)$ with $\mu_x = 0.04$, $\mu_y = 0.14$, and $\delta = 0.02$, determine the actuarial present value of a continuous annuity issued to $(x)$ and $(y)$ that pays 7,500 per year until the last of the death of $(x)$ and $(y)$.

$$APV = 7500 \cdot \overline{a}_{xy} = 7500 \cdot \left( \overline{a}_x + \overline{a}_y - \overline{a}_{xy} \right)$$

By CF, $\overline{a}_x = \frac{1}{\mu_x + \delta} = \frac{1}{0.06}, \overline{a}_y = \frac{1}{\mu_y + \delta} = \frac{1}{0.16}$, and by independence $\overline{a}_{xy} = \frac{1}{\mu_x + \mu_y + \delta} = \frac{1}{0.20}$

$$\therefore APV = 134,375$$
4. Under certain actuarial assumptions, you are given:

(i) \( \alpha(\infty) = 1.00076 \)

(ii) \( \beta(\infty) = 0.51627 \)

(iii) \( a_x = 5.439 \)

Using the UDD assumption, determine \( \bar{a}_x \)

By UDD, \( \bar{a}_x = \alpha(\infty) \cdot \bar{a}_x - \beta(\infty) \)

\( \bar{a}_x = 1 + a_x = 6.439 \)

\[ \therefore \bar{a}_x = 5.92762 \cdots \]

5. Using the actuarial assumptions in the Standard Sickness-Death Model in the L-TAM Tables, determine the APV of a 10-year deferred continuous annuity issued to a healthy 50-year old that pays 5,000 per year while the annuitant is healthy.

\[
APV = 5000 \cdot 10 \bar{a}_{50}^{00} = 5000 \cdot \left( 10 \bar{a}_{50}^{00} \cdot \bar{a}_{60}^{00} + 10 \bar{a}_{50}^{01} \cdot \bar{a}_{60}^{10} \right)
\]

\[
= 5000 \cdot \left( (1.05)^{-10} \cdot 10 \bar{a}_{50}^{00} \cdot \bar{a}_{60}^{00} + (1.05)^{-10} \cdot 10 \bar{a}_{50}^{01} \cdot \bar{a}_{60}^{10} \right)
\]

\[ \therefore APV = 21,640 \]