MAP 4176 / 5178 Name:_____
Test 3 Date: January 30, 2019

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

For Numbers 1 and 2, determine the APV of the annuity described, using constant force actuarial assumptions with $\mu = 0.02$ and $\delta = 0.04$.

1. a continuous 10-year temporary annuity with annual payment rate of 500 issued to (45)

$APV = 500 \cdot \bar{a}_{45:\overline{10}|} = 500 \cdot \left(\bar{a}_{45} - {}_{10}E_{45} \cdot \bar{a}_{55}\right)$

Since CF, $\bar{a}_{45} = \bar{a}_{55} = \frac{1}{\mu+\delta} = \frac{1}{0.06}$ and ${}_{10}E_{45} = e^{-10(\mu+\delta)} = e^{-0.6}$

$\therefore APV = 3{,}760$

2. a continuous 20-year certain-and-life annuity paying 3000 per year issued to (60)

$APV = 3000 \cdot \bar{a}_{\overline{60:20|}} = 3000 \cdot \left(\bar{a}_{\overline{20|}} + {}_{20}E_{60} \cdot \bar{a}_{80}\right)$

$\bar{a}_{\overline{20|}} = \frac{1-v^{20}}{\delta} = \frac{1-e^{-20(0.04)}}{0.04}$, ${}_{20}E_{60} = e^{-20(0.06)}$, and $\bar{a}_{80} = \frac{1}{0.06}$

$\therefore APV = 56{,}360$

3. Given independent lives (x) and (y) with $\mu_x = 0.04$, $\mu_y = 0.14$, and $\delta = 0.02$, determine the actuarial present value of a continuous annuity issued to (x) and (y) that pays 7,500 per year until the last of the death of (x) and (y).

$APV = 7500 \cdot \bar{a}_{\overline{xy}} = 7500 \cdot \left(\bar{a}_x + \bar{a}_y - \bar{a}_{xy}\right)$

By CF, $\bar{a}_x = \frac{1}{\mu_x+\delta} = \frac{1}{0.06}$, $\bar{a}_y = \frac{1}{\mu_y+\delta} = \frac{1}{0.16}$, and by independence $\bar{a}_{xy} = \frac{1}{\mu_x+\mu_y+\delta} = \frac{1}{0.20}$

$\therefore APV = 134{,}375$

4. Under certain actuarial assumptions, you are given:

   (i) $\alpha(\infty) = 1.00076$

   (ii) $\beta(\infty) = 0.51627$

   (iii) $a_x = 5.439$

   Using the UDD assumption, determine $\bar{a}_x$

   By UDD, $\bar{a}_x = \alpha(\infty) \cdot \ddot{a}_x - \beta(\infty)$

   $\ddot{a}_x = 1 + a_x = 6.439$

   $$\therefore \bar{a}_x = 5.92762\cdots$$

5. Using the actuarial assumptions in the Standard Sickness-Death Model in the L-TAM Tables, determine the APV of a 10-year deferred continuous annuity issued to a healthy 50-year old that pays 5,000 per year while the annuitant is healthy.

   $$APV = 5000 \cdot {}_{10|}\bar{a}^{00}_{50} = 5000 \cdot \left({}_{10}E^{00}_{50} \cdot \bar{a}^{00}_{60} + {}_{10}E^{01}_{50} \cdot \bar{a}^{10}_{60}\right)$$

   $$= 5000 \cdot \left((1.05)^{-10} \cdot {}_{10}p^{00}_{50} \cdot \bar{a}^{00}_{60} + (1.05)^{-10} \cdot {}_{10}p^{01}_{50} \cdot \bar{a}^{10}_{60}\right)$$

   $$\therefore APV = 21{,}640$$