MAP 4176 / 5178 Name:____________________

Test 4 Date: February 8, 2017

Show all work for full credit, use correct notation., and clearly mark your answer.

1. A 5-year endowment insurance with a benefit of 5000 payable at the moment of death is issued to (50). Using ILT actuarial assumptions and the claims acceleration approach, determine the variance of the present value random variable for this insurance.

$$Z = 5000 \cdot \bar{Z}_{50:\overline{5}|} \Rightarrow Var(Z) = 5000^2 \left({}^2\bar{A}_{50:\overline{5}|} - (\bar{A}_{50:\overline{5}|})^2 \right)$$

$$\bar{A}_{50:\overline{5}|} = \bar{A}^{1}_{50:\overline{5}|} + {}_5E_{50} \overset{CAA}{=} (1+i)^{1/2} \cdot A^{1}_{50:\overline{5}|} + {}_5E_{50} = (1+i)^{1/2}\left[A_{50} - {}_5E_{50} \cdot A_{55}\right] + {}_5E_{50}$$

$$\Rightarrow \bar{A}_{50:\overline{5}|} \overset{ILT}{=} .75115\cdots$$

$${}^2\bar{A}_{50:\overline{5}|} \overset{CAA}{=} (1+2i+i^2)^{1/2} \cdot \left[{}^2A_{50} - {}^2_5E_{50} \cdot {}^2A_{55}\right] + {}^2_5E_{50} \overset{ILT}{=} .56483\cdots$$

$$\therefore Var(Z) = 5000^2 \left(.56483\cdots - (.75115\cdots)^2 \right) = 14883.59$$

2. A 10-year deferred whole-life insurance with a benefit of 1000 payable at the moment of death is issued to (30). Using ILT mortality and $i = 0.06$, and assuming a uniform distribution of deaths between integer ages, determine the variance of the present value random variable for this insurance.

$$Z = 1000 \cdot {}_{10|}\bar{Z}_{30} \Rightarrow Var(Z) = 1000^2 \left({}^2_{10|}\bar{A}_{30} - ({}_{10|}\bar{A}_{30})^2 \right)$$

$${}_{10|}\bar{A}_{30} = {}_{10}E_{30} \cdot \bar{A}_{40} \overset{UDD}{=} {}_{10}E_{30} \cdot \frac{i}{\delta} \cdot A_{40} \overset{ILT}{=} .09091\ldots$$

$${}^2_{10|}\bar{A}_{30} \overset{UDD}{=} {}^2_{10}E_{30} \cdot \frac{2i+i^2}{2\delta} \cdot {}^2A_{40} \overset{ILT}{=} .02109\cdots$$

$$\therefore Var(Z) = 1000^2 \left(.02109\cdots - (.09091\cdots)^2 \right) = 12828.64$$

3. A 30-year term insurance with benefit payable at the moment of death is issued to (40). The death benefit paid at time t is $b_t = (1.06)^t$. Assuming the age at death random variable is uniformly distributed from 0 to 100, determine the actuarial present value of this insurance using $i = 0.06$. mortality is DML ($\omega = 100$)

$$APV = \int_0^{30} \underbrace{b_t \cdot v^t}_{=1} \cdot \underbrace{{}_tp_{40} \cdot \mu_{40}(t)}_{\overset{DML}{=} q_{40} \overset{\omega=100}{=} \frac{1}{60}} \cdot dt$$

$$\therefore APV = \int_0^{30} \frac{1}{60}\, dt = \frac{1}{60} \cdot t \Big|_0^{30} = \frac{30}{60} = 0.5$$

4. Using ILT actuarial assumptions, determine the actuarial present value of a 20-year temporary life annuity immediate issued to (35) with annual payments of 5000.

$$Y = 5000\, Y_{35:\overline{20}|} \Rightarrow APV = E[Y] = 5000 \cdot a_{35:\overline{20}|}$$

$$a_{35:\overline{20}|} = \ddot{a}_{35:\overline{20}|} - 1 + {}_{20}E_{35}$$

$$\ddot{a}_{35:\overline{20}|} = \ddot{a}_{35} - {}_{20}E_{35} \cdot \ddot{a}_{55} \overset{ILT}{=} 11.8817\ldots$$

$$\Rightarrow a_{35:\overline{20}|} \overset{ILT}{=} 11.1677\ldots$$

$$\therefore APV = 5000(11.1677\ldots) = 55{,}838.61$$

5. Let C denote the expected present value of a 10-year certain-and-life annuity due with annual payments of X issued to (50), and let L denote the expected present value of a life annuity due with annual payments of 10,000 also issued to (50). Using ILT mortality and $i = 0.06$, determine the value of X for which $C = L$.
(X is the actuarially equivalent 10-yr C&L benefit to the 10,000 whole-life benefit.)

$$C = X \cdot \ddot{a}_{\overline{50:\overline{10}|}} = X \cdot (\ddot{a}_{\overline{10}|} + {}_{10|}\ddot{a}_{50}) = X \cdot (\ddot{a}_{\overline{10}|} + {}_{10}E_{50} \cdot \ddot{a}_{60}) \overset{ILT}{=} X \cdot (13.4948\ldots)$$

$$L = 10000 \cdot \ddot{a}_{50} \overset{ILT}{=} 132668$$

$$C = L \Rightarrow X = \frac{132668}{13.4948\ldots} = 9830.99$$