MAP 4176 / 5178 Name:________________________

Test 4 Date: February 14, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. Use ILT actuarial assumptions and the claims acceleration approach to approximate the actuarial present value of a whole life insurance of 100,000 issued to (50) with benefit payable at the end of the quarter of death.

$$APV = 100000\, A_{50}^{(4)} \overset{CAA}{=} 100000(1+i)^{3/8} \cdot A_{50} \overset{ILT}{=} 25{,}455.184\ldots$$

2. Use ILT actuarial assumptions and assume a uniform distribution of deaths between integer ages to calculate the actuarial present value of a 20-year deferred whole life insurance of 50,000 issued to (30) with benefit payable at the end of the semiannual period of death.

$$APV = 50000 \cdot {}_{20|}A_{30}^{(2)} = 50000\, {}_{20}E_{30} \cdot A_{50}^{(2)}$$

$$A_{50}^{(2)} \overset{UDD}{=} \frac{i}{i^{(2)}} \cdot A_{50}$$

$$\therefore APV \overset{ILT}{=} 3711.859\ldots$$

3. Use ILT actuarial assumptions and assume a uniform distribution of deaths between integer ages to calculate the APV of a 30-year endowment insurance of 10,000 issued to (25) where the death benefit is paid at the moment of death.

$$APV = 10000 \cdot \bar{A}_{25:\overline{30|}} = 10000\left(\bar{A}^{1}_{25:\overline{30|}} + {}_{30}E_{25}\right)$$

$${}_{30}E_{25} = {}_{10}E_{25} \cdot {}_{20}E_{35} \quad \text{(among many correct ways)}$$

$$\bar{A}^{1}_{25:\overline{30|}} \overset{UDD}{=} \frac{i}{\delta} \cdot A^{1}_{25:\overline{30|}} = \frac{i}{\delta}\left(A_{25} - {}_{30}E_{25} \cdot A_{55}\right)$$

$$\therefore APV \overset{ILT}{=} 1919.453\ldots$$

4. A 10-year deferred whole life insurance of 25,000 is issued to a 40-year old. The benefit is payable at the moment of death. Determine the APV of the insurance policy using an annual effective interest rate of 5% and $DML(\omega = 100)$ mortality.

$$APV = 25000 \cdot {}_{10|}\bar{A}_{40} = 25000 \cdot {}_{10}E_{40} \cdot \bar{A}_{50}$$

$${}_{10}E_{40} = v^{10} \cdot {}_{10}P_{40} = (1.05)^{-10} \cdot \frac{100-40-10}{100-40}$$

$$\bar{A}_{50} = \frac{1}{100-50} \cdot \bar{a}_{\overline{100-50|}} = .02 \cdot \frac{1-v^{50}}{\delta}$$

$$\delta = \ln(1.05)$$

$$\therefore\ APV = 4785.610\ldots$$

5. A 10-year deferred, 20-year term, continuous insurance of 30,000 is issued to (x). In particular, a death benefit of 30,000 is paid at the moment of death if (x) dies between the ages of $x + 10$ and $x + 30$. Determine the actuarial present value of this insurance policy assuming $\mu = 0.02$ and $\delta = 0.04$.

$$APV = 30000 \cdot {}_{10}E_x \cdot \bar{A}^{\,1}_{x+10:\overline{20|}}$$

$${}_{10}E_x = v^{10} \cdot {}_{10}P_x \stackrel{CF}{=} e^{-10(\mu+\delta)}$$

$$\bar{A}^{\,1}_{x+10:\overline{20|}} = \bar{A}_{x+10} - {}_{20}E_{x+10} \cdot \bar{A}_{x+30}$$

$$\left.\begin{array}{l} \bar{A}_{x+10} \stackrel{CF}{=} \bar{A}_{x+30} \stackrel{CF}{=} \dfrac{\mu}{\mu+\delta} \\ {}_{20}E_{x+10} \stackrel{CF}{=} e^{-20(\mu+\delta)} \end{array}\right\} \Rightarrow \bar{A}^{\,1}_{x+10:\overline{20|}} = \frac{\mu}{\mu+\delta}\left(1-e^{-20(\mu+\delta)}\right)$$

$$\therefore\ APV = 3835.127\ldots$$