Solutions to Tests 5 & 6 Takehome Exam

1) $Z = 1000 Z_{50}$

$Var(Z) = 1000^2 \cdot ({}^2A_{50} - (A_{50})^2) = 32,734.097\cdots$

2)

K : Z	P_r
0 : $2000v$	$q_{35} = \frac{1}{60}$
1 : $2000v^2$	$p_{35} = \frac{59}{60}$

$E[Z] = 2000v \cdot \frac{1}{60} + 2000v^2 \cdot \frac{59}{60}$

$E[Z^2] = (2000v)^2 \cdot \frac{1}{60} + (2000v^2)^2 \cdot \frac{59}{60}$

$\therefore Var(Z) = 134.831\cdots$

3) $Z = 3000 \cdot {}_{20|}Z_{30}^{(4)} \Rightarrow Var(Z) = 3000^2 \cdot \left({}^2_{20|}A_{30}^{(4)} - \left({}_{20|}A_{30}^{(4)}\right)^2\right)$

${}_{20|}A_{30}^{(4)} = {}_{20}E_{30} \cdot A_{50}^{(4)} \overset{CAA}{=\!=} {}_{20}E_{30} \cdot (1+i)^{3/8} \cdot A_{50} \overset{ILT}{=} .07477\cdots$

${}^2_{20|}A_{30}^{(4)} = {}^2_{20}E_{30} \cdot {}^2A_{50}^{(4)} = {}^2_{20}E_{30} \cdot (1+2i+i^2)^{3/8} \cdot {}^2A_{50}$

$\vdots$

$\overset{ILT}{=} .00906\cdots$

$\therefore Var(Z) \overset{ILT}{=} 31282.739\cdots$

4) $Z = 4000 \cdot \bar{Z}_{30:\overline{20}|}$

$Var(Z) = 4000^2 \cdot \left({}^2\bar{A}_{30:\overline{20}|} - (\bar{A}_{30:\overline{20}|})^2 \right)$

$\bar{A}_{30:\overline{20}|} = \bar{A}^{1}_{30:\overline{20}|} + {}_{20}E_{30} \overset{UDD}{=} \frac{i}{\delta} \cdot (A_{30} - {}_{20}E_{30} \cdot A_{50}) + {}_{20}E_{30}$

$\therefore {}^2\bar{A}_{30:\overline{20}|} = \frac{2i+i^2}{2\delta} \cdot ({}^2A_{30} - {}^{2}_{20}E_{30} \cdot {}^2A_{50}) + {}^{2}_{20}E_{30}$

$\therefore Var(Z) \overset{ILT}{=} 68,708.788\ldots$

5) $E[Z] = 5000 \cdot {}_{10}E_{30} \cdot A_{40:\overline{10}|} = 5000 \cdot {}_{10}E_{30} \cdot (A_{40} - {}_{10}E_{40} \cdot A_{50})$

$E[Z^2] = 5000^2 \cdot {}^{2}_{10}E_{30} \cdot ({}^2A_{40} - {}^{2}_{10}E_{40} \cdot {}^2A_{50})$

$\therefore Var(Z) \overset{ILT}{=} 148,861.938\ldots$

6) $Z = 6000\ Z_{\overline{40:40}:\overline{10}|}$

$Var(Z) = 6000^2 \cdot \left({}^2\bar{A}_{\overline{40:40}:\overline{10}|} - (A_{\overline{40:40}:\overline{10}|})^2 \right)$

$A_{\overline{40:40}:\overline{10}|} = 2 \cdot A_{40:\overline{10}|} - A_{40:40:\overline{10}|}$

$\Rightarrow A_{\overline{40:40}:\overline{10}|} = 2 \cdot (A_{40} - {}_{10}E_{40} \cdot A_{50} + {}_{10}E_{40}) - [A_{40:40} - {}_{10}E_{40:40} \cdot A_{50:50} + {}_{10}E_{40:40}]$

$\therefore {}^2A_{\overline{40:40}:\overline{10}|} = 2 \cdot ({}^2A_{40} - {}^{2}_{10}E_{40} \cdot {}^2A_{50} + {}^{2}_{10}E_{40}) - [{}^2A_{40:40} - {}^{2}_{10}E_{40:40} \cdot {}^2A_{50:50} + {}^{2}_{10}E_{40:40}]$

$\therefore Var(Z) \overset{ILT}{=} 1020.115\ldots$

7) $Y = 100\,\bar{Y}_x = 100 \cdot \frac{1-\bar{Z}_x}{\delta}$

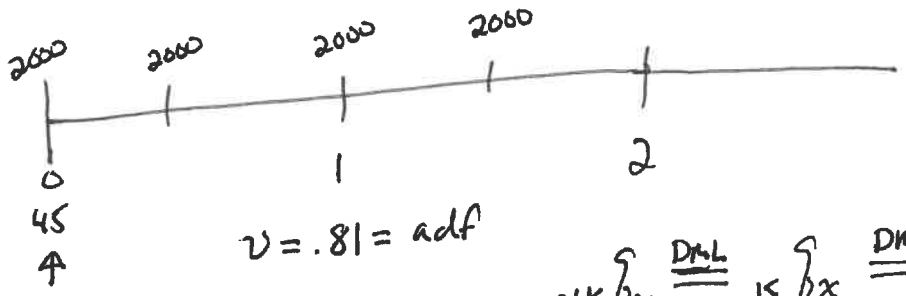
$Var(Y) = 100^2 \cdot \frac{{}^2\bar{A}_x - (\bar{A}_x)^2}{\delta^2}$

$\bar{A}_x \overset{CF}{=\!=} \frac{\mu}{\mu+\delta}$

${}^2\bar{A}_x \overset{CF}{=\!=} \frac{\mu}{\mu+2\delta}$

$\therefore Var(Y) = 1{,}000{,}000$

8)



$v = .81 = adf$

$nlK\,{}_t q_x \overset{DML}{=\!=} {}_K q_x \overset{DML}{=\!=} \frac{K}{\omega - x}$

Y	Pr
2000	${}_{.5}q_{45} = .01$
$3860 = 2000(1+v^{1/2})$	${}_{.5|.5}q_{45} = .01$
$5420 = 2000(1+v^{1/2}+v)$	${}_{1|.5}q_{45} = .01$
$6878 = 2000(1+v^{1/2}+v+v^{3/2})$	${}_{1.5}p_{45} = .97$

$\therefore E[Y] = .01(2000 + 3800 + 5420) + .97(6878)$

$E[Y^2] = .01(2000^2 + 3800^2 + 5420^2) + .97(6878^2)$

$\Rightarrow Var(Y) = 345{,}084.980\ldots$

9) $Y = 3600 \cdot \ddot{Y}^{(12)}_{30:\overline{10}|} = 3600 \cdot \frac{1 - Z^{(12)}_{30:\overline{10}|}}{d^{(12)}}$

$\therefore Var(Y) = 3600^2 \cdot \frac{{}^2A^{(12)}_{30:\overline{10}|} - (A^{(12)}_{30:\overline{10}|})^2}{(d^{(12)})^2}$

$A^{(12)}_{30:\overline{10}|} \overset{UDD}{=} \frac{i}{i^{(12)}} \cdot (A_{30} - {}_{10}E_{30} \cdot A_{40}) + {}_{10}E_{30}$

${}^2A^{(12)}_{30:\overline{10}|} \overset{UDD}{=} \frac{2i+i^2}{2i^{(12)}} \cdot ({}^2A_{30} - {}^2_{10}E_{30} \cdot {}^2A_{40}) + {}^2_{10}E_{30}$

$\hookrightarrow {}^2i^{(12)}: \quad v = \frac{1}{1+i} = \frac{1}{(1+\frac{i^{(12)}}{12})^{12}}$

$\Rightarrow v^2 = \frac{1}{(1+i)^2} = \frac{1}{(1+\frac{i^{(12)}}{12})^{24}} = \frac{1}{(1+\frac{2i^{(12)}}{12})^{12}}$

$\therefore (1+\frac{2i^{(12)}}{12})^{12} = (1+i)^2$

$\Rightarrow {}^2i^{(12)} = 12 \cdot [(1+i)^{1/6} - 1]$

$\therefore Var(Y) \overset{ILT}{=} 3,530,890.007\ldots$

10) $Y = 400 \cdot \ddot{Y}_{30:40} = 400 \cdot \frac{1 - Z_{30:40}}{d}$

$Var(Y) = 400^2 \cdot \frac{{}^2A_{30:40} - (A_{30:40})^2}{d^2}$

$\therefore Var(Y) = 1,416,569.681\ldots$