MAP 4176 / 5178 Name: KEY
Test 5 Date: February 24, 2016

Show all work for full credit, use correct notation., and clearly mark your answer.

For numbers 1 and 2, you are given:

a. $i = 0.05$
b. $\ddot{a}_{50} = 15$
c. ${}_{10}E_{40} = 0.6$

1. Determine the actuarial present value of a 10-year deferred whole life annuity due on (40) with benefit equal to 5000 per month. Use the UDD assumption and note that for $i = 0.05$, $\alpha(12) = 1.000197$ and $\beta(12) = 0.46651$.

$$APV = 12(5000) \cdot {}_{10|}\ddot{a}_{40}^{(12)} = 60000 \cdot {}_{10}E_{40} \cdot \ddot{a}_{50}^{(12)}$$

$$\ddot{a}_{50}^{(12)} = \alpha(12) \cdot \ddot{a}_{50} - \beta(12) = 1.000197(15) - 0.46651$$

$$\therefore \; APV = 523{,}312.02$$

2. Determine the actuarial present value of a 10-year deferred whole life annuity due on (40) with benefit equal to 5000 per month using the two-term Woolhouse Formula.

$$APV = 12(5000) \cdot {}_{10|}\ddot{a}_{40}^{(12)} = 60000 \; {}_{10}E_{40} \; \ddot{a}_{50}^{(12)}$$

$$\ddot{a}_{50}^{(12)} \overset{\text{2-term}}{\underset{\text{WH}}{=}} \ddot{a}_{50} - \frac{11}{24} = 15 - \frac{11}{24}$$

$$\therefore \; APV = 523{,}500$$

3. Given $\bar{a}_x = 13$, $\bar{a}_{x:\overline{n}|} = 7$, and ${}_nE_x = 0.6$, determine $\bar{a}_{x+n}$.

$$\bar{a}_x = \bar{a}_{x:\overline{n}|} + {}_nE_x \cdot \bar{a}_{x+n}$$

$$13 = 7 + 0.6 \cdot \bar{a}_{x+n} \Rightarrow \bar{a}_{x+n} = 10$$

4. Use constant force assumptions with $\delta = 0.03$ and $\mu = 0.02$ to determine the expected present value of a continuous 10-year temporary annuity issued to (x) paying at a rate of 1000 per year.

$$EPV = 1000\, \bar{a}_{x:\overline{10}|} = 1000\left(\bar{a}_x - {}_{10}E_x \cdot \bar{a}_{x+10}\right)$$

$$\bar{a}_x = \int_0^\infty v^t \cdot {}_tp_x \, dt = \int_0^\infty e^{-\delta t} \cdot e^{-\mu t} \, dt = \int_0^\infty e^{-(\mu+\delta)t} \, dt = \frac{1}{\mu+\delta}$$

$$\bar{a}_{x+10} = \frac{1}{\mu+\delta} \text{ also.} \qquad {}_{10}E_x = v^{10} \cdot {}_{10}p_x = e^{-10\delta} \cdot e^{-10\mu} = e^{-10(\mu+\delta)}$$

$$\therefore EPV = 1000\left(\frac{1}{\mu+\delta} - e^{-10(\mu+\delta)} \cdot \frac{1}{\mu+\delta}\right) = 1000 \cdot \frac{1}{\mu+\delta}\left(1 - e^{-10(\mu+\delta)}\right)$$

$$\therefore EPV = 1000 \cdot \frac{1}{.05}\left(1 - e^{-.5}\right) = 7869.39$$

Note $\bar{a}_{x:\overline{10}|} = \int_0^{10} v^t \cdot {}_tp_x \, dt$ also

5. Use constant force assumptions with $\delta = 0.03$ and $\mu = 0.02$ to determine the expected present value of a continuous 10-year certain and life annuity issued to (x) paying at a rate of 1000 per year.

$$EPV = 1000\, \bar{a}_{\overline{x:\overline{10}|}} = 1000\left(\bar{a}_{\overline{10}|} + {}_{10}E_x\, \bar{a}_{x+10}\right)$$

$${}_{10}E_x = e^{-10(\mu+\delta)} \qquad \bar{a}_{\overline{10}|} = \frac{1 - e^{-10\delta}}{\delta} = \frac{1 - e^{-.3}}{.03}$$

$$\bar{a}_{x+10} = \frac{1}{\mu+\delta}$$

$$\therefore EPV = 1000\left(\frac{1 - e^{-.3}}{.03} + e^{-.5} \cdot \frac{1}{.05}\right) = 20770$$