MAP 4176 / 5178 Name:________________________

Test 5 Date: February 15, 2017

Show all work for full credit, use correct notation., and clearly mark your answer. Each problem is worth 10 points.

1. You are given:

   $(I\ddot{a})_{x:\overline{10|}} = 33.6220$

   $(Ia)_{x:\overline{10|}} = 30.6127$

   $\ddot{a}_{x:\overline{10|}} = 7.0318$

   $i = 0.06$

   Determine ${}_{10}p_x$

$$(I\ddot{a})_{x:\overline{10|}} = (Ia)_{x:\overline{10|}} + \ddot{a}_{x:\overline{10|}} - 10 \cdot {}_{10}E_x$$

$$\Rightarrow 33.622 = 30.6127 + 7.0318 - 10(1.06)^{-10} \cdot {}_{10}p_x$$

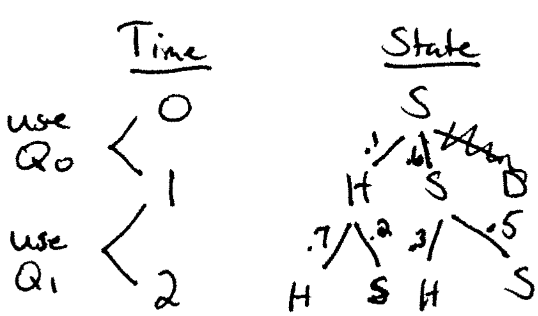
$$\Rightarrow {}_{10}p_x = .7203\ldots$$

2. For a non-homogeneous discrete-time Markov model for an insured population, Q_i denotes the annual transition probability matrix from time i to time $i+1$. You are given:

Q_0	Healthy	Sick	Terminated
Healthy	0.5	0.3	0.2
Sick	0.1	0.6	0.3
Terminated	0.0	0.0	1.0

Q_k $(k = 1, 2, \ldots)$	Healthy	Sick	Terminated
Healthy	0.7	0.2	0.1
Sick	0.3	0.5	0.2
Terminated	0.0	0.0	1.0

A 3-year temporary annuity pays 800 at the beginning of each year that an insured is not in the terminated state. Using $v = 0.9$, determine the actuarial present value of this annuity for an insured who is initially sick.



$$Pr(H \text{ or } S \text{ @ } t=1) = .1 + .6 = .7$$

$$Pr(H \text{ or } S \text{ @ } t=2) = (.1)(.7) + (.1)(.2) + (.6)(.3) + (.6)(.5) = .57$$

$$\therefore \text{APV} = 800 + 800v(.7) + 800v^2(.57) \overset{v=.9}{=\!=} 1673.36$$

3. An insurance company sells life annuities to 30-year olds with non-level annual payments. The first payment for these annuities is 1000, paid at age 40, and subsequent payments increase by 2%, up to a maximum of 20 payments. Using ILT mortality and $i = 8.12\%$, determine the single net premium for these annuities. $v = \frac{1}{1.0812}$

$$SNP = 1000 v^{10} \cdot {}_{10}P_{30} + 1000(1.02) \cdot v^{11} \cdot {}_{11}P_{30} + \cdots + 1000(1.02)^{19} \cdot v^{29} \cdot {}_{29}P_{30}$$

$$= 1000 v^{10} \cdot {}_{10}P_{30} \underbrace{\left(1 + \frac{1}{1.06} P_{40} + \cdots + \left(\frac{1}{1.06}\right)^{19} \cdot {}_{19}P_{40}\right)}_{= \ddot{a}_{40:\overline{20}|.06}}$$

$$\therefore\ SNP = 1000\left(\frac{1}{1.0812}\right)^{10} \cdot \frac{l_{40}}{l_{30}} \cdot \left(\ddot{a}_{40} - {}_{20}E_{40} \cdot \ddot{a}_{60}\right) \stackrel{ILT}{=} 5280.826\ldots$$

4. You are given $\ddot{a}_{65} = 10$ and $\ddot{a}_{65:65} = 8$.

For a couple, both age 65, determine the expected present value of an annuity that pays 3000 while both are alive, increasing to 5000 to the husband upon the death of the wife, but decreasing to 2000 to the wife upon the death of the husband.

$$EPV = 3000 \cdot \ddot{a}_{65:65} + 5000\left(\ddot{a}_{65} - \ddot{a}_{65:65}\right) + 2000\left(\ddot{a}_{65} - \ddot{a}_{65:65}\right)$$

$$= 3000(8) + 5000(2) + 2000(2) = 38{,}000$$

5. You are given:

Mortality follows Gompertz's Law: $\mu_x = B \cdot c^x$ with $B = 0.00004$ and $c = 1.1$
$i = 0.06$
$\ddot{a}_{65} = 9.9$

A retirement benefit pays 5000 at the beginning of each month to (65) for life. Let U denote the actuarial present value of the retirement benefit using the UDD assumption, and let W denote the actuarial present value of the retirement benefit using the three-term Woolhouse formula. Determine $W - U$.

$$APV = 12(5000) \cdot \ddot{a}^{(12)}_{65}$$

$$U = 60000 \cdot \left(\alpha(12) \cdot \ddot{a}_{65} - \beta(12)\right) = 60000\left(1.00028(9.9) - .46812\right)$$

$$\Rightarrow U = 566{,}079.12$$

$$W = 60000\left(\ddot{a}_{65} - \frac{11}{24} - \frac{143}{12^3}\left(\ln(1.06) + \mu_{65}\right)\right) \quad \mu_{65} = .00004(1.1)^{65}$$

$$\Rightarrow W = 566{,}113.285\ldots$$

$$\therefore\ W - U = 34.165\ldots$$