MAP 4176 / 5178 Name: KEY

Test 6 Date: March 2, 2016

Show all work for full credit, use correct notation., and clearly mark your answer.

1. You are given:

   (i) $\bar{a}_x = 8$

   (ii) $\bar{a}_y = 10$

   (iii) $\bar{A}_{xy} = 0.8$

   (iv) $\delta = 0.05$

   Determine $\bar{A}_{\overline{xy}}$.

Solution: $\bar{A}_x = 1 - \delta \bar{a}_x = 1 - .05(8) = .6$

$\bar{A}_y = 1 - \delta \bar{a}_y = 1 - .05(10) = .5$

$\Rightarrow \bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.6 + 0.5 - 0.8 = 0.3$

Alternatively: $\bar{a}_{xy} = \dfrac{1 - \bar{A}_{xy}}{\delta} = \dfrac{1 - .8}{.05} = 4$

$\Rightarrow \bar{a}_{\overline{xy}} = \bar{a}_x + \bar{a}_y - \bar{a}_{xy} = 8 + 10 - 4 = 14$

$\therefore \bar{A}_{\overline{xy}} = 1 - \delta \bar{a}_{\overline{xy}} = 1 - .05(14) = 0.3$

2. You are given:

   (i) $A^{1}_{x:\overline{n}|} = 0.125$

   (ii) ${}_nE_x = 0.7$

   (iii) $i = 0.04$

   Determine $\ddot{a}_{x:\overline{n}|}$

$A_{x:\overline{n}|} = A^{1}_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ 1} = A^{1}_{x:\overline{n}|} + {}_nE_x = .125 + .7$

$\therefore A_{x:\overline{n}|} = .825$

$\ddot{a}_{x:\overline{n}|} = \dfrac{1 - A_{x:\overline{n}|}}{d} = \dfrac{1 - .825}{.04}(1.04) = 4.55$

3. You are given:

(i) $\ddot{a}^{(4)}_{x:\overline{20|}} = 16.5$

(ii) ${}_{20}p_x = 0.6$

(iii) $d = 0.04$

Determine $A^{1\,(4)}_{x:\overline{20|}}$

$$\ddot{a}^{(4)}_{x:\overline{20|}} = \frac{1 - A^{(4)}_{x:\overline{20|}}}{d^{(4)}}$$

$$\Longleftrightarrow A^{(4)}_{x:\overline{20|}} = 1 - d^{(4)} \cdot \ddot{a}^{(4)}_{x:\overline{20|}}$$

$$1-d = \left(1 - \frac{d^{(4)}}{4}\right)^4 \Rightarrow d^{(4)} = 4\left[1-(.96)^{1/4}\right]$$

$$\therefore A^{(4)}_{x:\overline{20|}} = 1 - 4[1-(.96)^{1/4}](16.5) \qquad {}_{20}E_x = v^{20}\,{}_{20}p_x = (.96)^{20}(.6)$$

$$\therefore A^{1\,(4)}_{x:\overline{20|}} = A^{(4)}_{x:\overline{20|}} - {}_{20}E_x = 0.06466$$

4. You are given:

(i) $1000\,{}_{10}E_x = 507$

(ii) $1000\,{}_{20}E_{x+10} = 139$

(iii) $\ddot{a}_x = 13.08$

(iv) $\ddot{a}_{x+30} = 5.65$

(v) $i = 0.06$

Determine $\ddot{s}_{x:\overline{30|}}$.

$$\ddot{a}_{x:\overline{30|}} = \ddot{a}_x - {}_{30}E_x \cdot \ddot{a}_{x+30}$$

$$= \ddot{a}_x - {}_{10}E_x \cdot {}_{20}E_{x+10} \cdot \ddot{a}_{30}$$

$$\Longrightarrow \ddot{a}_{x:\overline{30|}} = 13.08 - (.507)(.139)(5.65) = 12.6818$$

$$\therefore \ddot{s}_{x:\overline{30|}} = \frac{\ddot{a}_{x:\overline{30|}}}{{}_{30}E_x} = \frac{12.6818}{(.507)(.139)} = 179.95$$

5. For a whole life annuity due with annual payments of 1000 issued to (x), you are given:

(i) Y denotes the present value random variable for this annuity

(ii) ${}^2A_x = 0.052$

(iii) $A_x = 0.169$

(iv) $i = 0.05$

Determine $Var(Y)$.

$$Y = 1000 \cdot \ddot{Y}_x = 1000\,\frac{1 - Z_x}{d} = 1000 \cdot \frac{1}{d}(1 - Z_x)$$

$$\therefore Var(Y) = 1000^2 \cdot \frac{1}{d^2} \cdot Var(Z_x) = 1000^2 \cdot \frac{1}{d^2}\left[{}^2A_x - (A_x)^2\right]$$

$$= \frac{1000^2}{(.05)^2}(1.05)^2\left[.052 - (.169)^2\right] = 10{,}336{,}599$$