MAP 4176 / 5178
Test 6

Name: KEY

Date: February 22, 2017

Show all work for full credit, use correct notation, and clearly mark your answer. Each problem is worth 10 points.

1. Use constant force assumptions with $\delta = 0.05$ and $\mu = 0.05$ to determine the expected present value of a continuous whole life annuity issued to (x) paying at a rate of 100 per year.

$$EPV = 100\cdot\bar{a}_x = 100\int_0^\infty v^t\cdot {}_tp_x\,dt = 100\int_0^\infty e^{-.05t}\cdot e^{-.05t}\,dt$$

$$= 100\int_0^\infty e^{-.1t}\,dt = \frac{100}{.1} = 1000$$

<u>Note</u>: $EPV = 100\cdot\frac{1}{\mu+\delta} = 1000$

2. Given $\bar{a}_x = 16$, $\bar{a}_{x:\overline{n}|} = 15$, $\bar{a}_{x+n} = 10$, and $v^n = 0.2$ determine ${}_np_x$.

$$\bar{a}_x = \bar{a}_{x:\overline{n}|} + {}_nE_x\cdot\bar{a}_{x+n} \implies {}_nE_x = \frac{\bar{a}_x - \bar{a}_{x:\overline{n}|}}{\bar{a}_{x+n}}$$

$$\implies {}_nE_x = v^n\cdot{}_np_x = .2\cdot{}_np_x = \frac{16-15}{10} \implies {}_np_x = .5$$

3. Given independent lives with $\mu_x = 0.05$, $\mu_y = 0.03$, and $\delta = 0.02$, determine the actuarial present value of a continuous 10-year temporary annuity paying at a rate of 5000 per year while both are alive.

$$APV = 5000\cdot\bar{a}_{xy:\overline{10}|} \qquad \mu_{xy} \overset{ind}{=} \mu_x + \mu_y = .08$$

$$\bar{a}_{xy:\overline{10}|} = \int_0^{10} v^t\cdot{}_tp_{xy}\,dt = \int_0^{10} e^{-.02t}\cdot e^{-.08t}\,dt$$

$$= \int_0^{10} e^{-.1t}\,dt = \frac{1}{.1}e^{-.1t}\Big|_{10}^{0} = 10(1-e^{-1})$$

$$\therefore APV = 5000\cdot 10\cdot(1-e^{-1}) = 31606.03$$

<u>Note</u>: $APV \overset{CF}{=} 5000\cdot\frac{1}{\mu_{xy}+\delta}\left(1 - {}_{10}E_{xy}\right)$

For numbers 4 and 5, you are to use the following information for a 4-state model with states 0, 1, 2, and 3 (state 3 is the dead state):

(i) $i = 0.05$

(ii) ${}_5p_{80}^{00} = 0.53880$ ${}_5p_{80}^{01} = 0.17327$ ${}_5p_{80}^{02} = 0.06956$

(iii) the following annuity values at 5% (all other annuity values are 0)

x	$\bar{a}_x^{00}$	$\bar{a}_x^{01}$	$\bar{a}_x^{02}$	$\bar{a}_x^{11}$	$\bar{a}_x^{10}$	$\bar{a}_x^{12}$	$\bar{a}_x^{22}$
80	5.5793	1.3813	0.6109	3.0936	1.6719	1.7206	4.4712
85	4.8066	1.0396	0.3403	2.6723	0.8834	1.0883	3.2367

4. A continuous annuity product issued to 80-year olds in state 0 pays as follows:

   8000 per year while in state 0 or in state 1
   30000 per year while in state 2

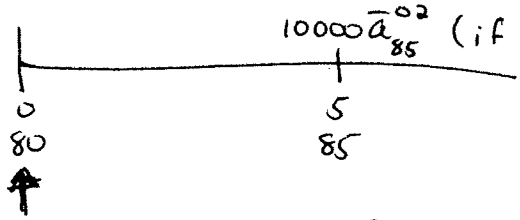
   Determine the net single premium for this annuity.

$$SNP = 8000\,\bar{a}_{80}^{00} + 8000\,\bar{a}_{80}^{01} + 30000\,\bar{a}_{80}^{02} = 74011.8$$

5. The company issuing the above annuity is considering adding a deferred feature in which an additional payment of 10000 per year is paid while in state 2, beginning at age 85. Determine the net additional cost of this annuity by adding this feature. (I.e. determine the increase in the net single premium).

$10000\,\bar{a}_{85}^{22}$ (if (85) is in state 2)
$10000\,\bar{a}_{85}^{12}$ (if (85) is in state 1)
$10000\,\bar{a}_{85}^{02}$ (if (85) is in state 0)

Net Additional Cost



$$\therefore EPV = 10000\,\bar{a}_{85}^{02}\cdot v^5\cdot {}_5p_{80}^{00} + 10000\,\bar{a}_{85}^{12}\cdot v^5\cdot {}_5p_{80}^{01} + 10000\,\bar{a}_{85}^{22}\cdot v^5\cdot {}_5p_{80}^{02}$$

$$= \frac{10000}{(1.05)^5}\left[(.3403)(.5388) + (1.0883)(.17327) + (3.2367)(.06956)\right]$$

$$\Rightarrow EPV = 4678.19$$