MAP 4176 / 5178 Name:________________________________

Test 6 Date: February 20, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation.

For Numbers 1 through 4, use the assumptions given in the problem to determine the variance of the random variable representing the present value of the benefit for the insurance product described.

1. a discrete 2-year endowment insurance of 10,000 issued to (60) using $DML(\omega = 100)$ mortality and $i = 6\%$

Z	Pr
$10000v$	$q_{60} = \frac{1}{40}$
$10000v^2$	$p_{60} = \frac{39}{40}$

$E[Z] = 10000v \cdot \frac{1}{40} + 10000v^2 \cdot \frac{39}{40}$

$E[Z^2] = (10000v)^2 \cdot \frac{1}{40} + (10000v^2)^2 \cdot \frac{39}{40}$

$$Var(Z) = E[Z^2] - (E[Z])^2 = 6951$$

2. a 20-year deferred continuous whole life insurance of 5000 issued to (30), using $CF(\mu = 0.02, \delta = 0.04)$ actuarial assumptions

$$Z = 5000 \cdot {}_{20|}\bar{Z}_{30} \;\Rightarrow\; Var(Z) = 5000^2 \cdot Var\left({}_{20|}\bar{Z}_{30}\right)$$

$$= 5000^2 \cdot \left({}^{2}_{20|}\bar{A}_{30} - \left({}_{20|}\bar{A}_{30}\right)^2\right)$$

$${}_{20|}\bar{A}_{30} = {}_{20}E_{30} \cdot \bar{A}_{50} = e^{-20(\mu+\delta)} \cdot \frac{\mu}{\mu+\delta}$$

$${}^{2}_{20|}\bar{A}_{30} = {}^{2}_{20}E_{30} \cdot {}^{2}\bar{A}_{50} = e^{-20(\mu+2\delta)} \cdot \frac{\mu}{\mu+2\delta}$$

$$\therefore Var(Z) = 424{,}682$$

3. a 10-year term discrete insurance of 50,000 issued to (20), using the L-TAM Table actuarial assumptions

$$Z = 50000 \cdot Z_{\overset{1}{20}:\overline{10|}} \quad \Rightarrow \quad Var(Z) = 50000^2 \cdot Var\left(Z_{\overset{1}{20}:\overline{10|}}\right)$$

$$= 50000^2 \cdot \left({}^2A_{\overset{1}{20}:\overline{10|}} - \left(A_{\overset{1}{20}:\overline{10|}}\right)^2\right)$$

Using the L-TAM Table, $A_{\overset{1}{20}:\overline{10|}} = A_{20:\overline{10|}} - {}_{10}E_{20}$ is the easiest way to calculate $A_{\overset{1}{20}:\overline{10|}}$ but this equation is not helpful when calculating ${}^2A_{\overset{1}{20}:\overline{10|}}$ since the only upper 2 values in the L-TAM Table are for whole life statuses. So we use

$$A_{\overset{1}{20}:\overline{10|}} = A_{20} - {}_{10}E_{20} \cdot A_{30}$$

$${}^2A_{\overset{1}{20}:\overline{10|}} = {}^2A_{20} - {}_{10}^{2}E_{20} \cdot {}^2A_{30} \quad \text{where } {}_{10}^{2}E_{20} = v^{10} \cdot {}_{10}E_{20}$$

$$\therefore Var(Z) = 4{,}068{,}300$$

4. a whole life insurance of 10,000 issued to (25), with benefit payable at the end of the quarter of death, using the L-TAM Table actuarial assumptions and the claims acceleration approach

$$Z = 10000 \cdot Z_{25}^{(4)} \quad \Rightarrow \quad Var(Z) = 10000^2 \cdot Var\left(Z_{25}^{(4)}\right)$$

$$= 10000^2 \cdot \left({}^2A_{25}^{(4)} - \left(A_{25}^{(4)}\right)^2\right)$$

$$A_{25}^{(4)} = (1+i)^{\frac{3}{8}} \cdot A_{25}$$

$${}^2A_{25}^{(4)} = (1+2i+i^2)^{\frac{3}{8}} \cdot {}^2A_{25} = (1+i)^{\frac{3}{4}} \cdot {}^2A_{25}$$

$$\therefore Var(Z) = 425{,}430$$

5. Use the L-TAM Table actuarial assumptions to calculate $Var(10000 \cdot v^K)$ where $K = K_{30}$

$$v^K = (1+i) \cdot v^{K+1} = (1+i) \cdot Z_{30}$$

$$\Rightarrow \quad Var(v^K) = (1+i)^2 \cdot Var(Z_{30}) = (1+i)^2 \cdot \left({}^2A_{30} - (A_{30})^2\right)$$

$$\therefore Var(10000 \cdot v^K) = 569{,}340$$