Each problem is worth 10 points. Show all work for full credit, and use correct notation.

For Numbers 1 through 4, use the assumptions given in the problem to determine the variance of the random variable representing the present value of the benefit for the insurance product described.

1. a discrete 2-year endowment insurance of 10,000 issued to (60) using $DML(\omega = 100)$ mortality and $i = 6$

<table>
<thead>
<tr>
<th>$Z$</th>
<th>$Pr$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10000v$</td>
<td>$q_{60} = \frac{1}{40}$</td>
</tr>
<tr>
<td>$10000v^2$</td>
<td>$p_{60} = \frac{39}{40}$</td>
</tr>
</tbody>
</table>

$E[Z] = 10000v \cdot \frac{1}{40} + 10000v^2 \cdot \frac{39}{40}$

$E[Z^2] = (10000v)^2 \cdot \frac{1}{40} + (10000v^2)^2 \cdot \frac{39}{40}$

$Var(Z) = E[Z^2] - (E[Z])^2 = 6951$

2. a 20-year deferred continuous whole life insurance of 5000 issued to (30), using $CF(\mu = 0.02, \delta = 0.04)$ actuarial assumptions

$Z = 5000 \cdot 20|\bar{Z}_{30} \Rightarrow Var(Z) = 5000^2 \cdot Var(20|\bar{Z}_{30})$

$= 5000^2 \cdot \left( \frac{2}{20|\bar{A}_{30}} - \left( \frac{2}{20|\bar{A}_{30}} \right)^2 \right)$

$20|\bar{A}_{30} = 20E_{30} \cdot \bar{A}_{50} = e^{-20(\mu+\delta)} \cdot \frac{\mu}{\mu+\delta}$

$20|\bar{A}_{30}^2 = 20E_{30} \cdot 2\bar{A}_{50} = e^{-20(\mu+2\delta)} \cdot \frac{\mu}{\mu+2\delta}$

$\therefore Var(Z) = 424,682$
3. a 10-year term discrete insurance of 50,000 issued to (20), using the L-TAM Table actuarial assumptions

\[
Z = 50000 \cdot \frac{1}{20:10} \Rightarrow Var(Z) = 50000^2 \cdot Var\left(\frac{1}{20:10}\right)
\]

\[
= 50000^2 \cdot \left(\frac{2}{20:10} - \left(\frac{1}{20:10}\right)^2\right)
\]

Using the L-TAM Table, \(A_{20:10} = A_{20:10} - 10E_{20}\) is the easiest way to calculate \(A_{20:10}\) but this equation is not helpful when calculating \(2A_{20:10}\) since the only upper 2 values in the L-TAM Table are for whole life statuses. So we use

\[
A_{20:10} = A_{20} - 10E_{20} \cdot A_{30}
\]

\[
2A_{20:10} = 2A_{20} - 10E_{20} \cdot 2A_{30} \text{ where } 10E_{20} = v^{10} \cdot 10E_{20}
\]

\[
\therefore Var(Z) = 4,068,300
\]

4. a whole life insurance of 10,000 issued to (25), with benefit payable at the end of the quarter of death, using the L-TAM Table actuarial assumptions and the claims acceleration approach

\[
Z = 10000 \cdot Z_{25}^{(4)} \Rightarrow Var(Z) = 10000^2 \cdot Var\left(Z_{25}^{(4)}\right)
\]

\[
= 10000^2 \cdot \left(\frac{4}{25} - \left(\frac{4}{25}\right)^2\right)
\]

\[
A_{25}^{(4)} = (1 + i)^{\frac{3}{5}} \cdot A_{25}
\]

\[
2A_{25}^{(4)} = (1 + 2i + i^2)^{\frac{3}{5}} \cdot 2A_{25} = (1 + i)^{\frac{3}{5}} \cdot 2A_{25}
\]

\[
\therefore Var(Z) = 425,430
\]

5. Use the L-TAM Table actuarial assumptions to calculate \(Var(10000 \cdot v^K)\) where \(K = K_{30}\)

\[
v^K = (1 + i) \cdot v^{K+1} = (1 + i) \cdot Z_{30}
\]

\[
\Rightarrow Var(v^K) = (1 + i)^2 \cdot Var(Z_{30}) = (1 + i)^2 \cdot \left(2A_{30} - (A_{30})^2\right)
\]

\[
\therefore Var(10000 \cdot v^K) = 569,340
\]