MAP 4176 / 5178 Name: KEY

Test 7 Date: March 23, 2016

Show all work for full credit, use correct notation., and clearly mark your answer.

1. A 20-year deferred whole life annuity due issued to (40) pays 60,000 at the beginning of each year. Premiums of 21,000 are paid at the beginning of each year during the deferred period. Using i = 0.06, determine the value of the loss-at-issue present value random variable if (40) dies at age 67.5

$$({}_0L \mid T = 27.5) = 60000\, \ddot{a}_{\overline{8}|}\, v^{20} - 21000\, \ddot{a}_{\overline{20}|}$$

$$= -132{,}175.39$$

2. For a fully discrete whole life insurance of 50,000 issued to (x) with annual premiums of 480, you are given

(a) $A_x = 0.141$
(b) $i = 0.06$

Determine the expected value of the loss-at-issue present value random variable

$$E[{}_0L] = 50000 A_x - 480\, \ddot{a}_x \qquad \ddot{a}_x = \frac{1 - A_x}{d} = 15.175\overline{6}$$

$$\therefore E[{}_0L] = -234.32$$

3. For a fully continuous whole life insurance of 1000 issued to (x) with annual premium rate of 42, use $CF(\mu = .04, \delta = .06)$ assumptions to determine the variance of the loss-at-issue present value random variable

$${}_0L = 1000\, \bar{Z}_x - 42\, \bar{Y}_x \qquad \bar{Y}_x = \frac{1 - \bar{Z}_x}{\delta}$$

$$\therefore {}_0L = \left(1000 + \frac{42}{.06}\right) \bar{Z}_x - \frac{42}{.06} = 1700\, \bar{Z}_x - 700$$

$$\Rightarrow Var({}_0L) = 1700^2 \left[{}^2\bar{A}_x - (\bar{A}_x)^2\right] \qquad \bar{A}_x = \frac{\mu}{\mu + \delta} = .4$$

$${}^2\bar{A}_x = \frac{\mu}{\mu + 2\delta} = .25$$

$$Var({}_0L) = 1700^2\left[.25 - (.4)^2\right] = 260{,}100$$

4. For a fully continuous whole life insurance of 1 issued to (x) with annual premium rate, π, you are given:

(a) π is determined using the equivalence principle.
(b) $\bar{A}_x = 0.6$ and $^2\bar{A}_x = 0.5$

Determine the variance of the loss-at-issue present value random variable.

$$_0L = \bar{Z}_x - \pi \bar{Y}_x \xrightarrow{\bar{Y}_x = \frac{1-\bar{Z}_x}{\delta}} \left(1+\frac{\pi}{\delta}\right)\bar{Z}_x - \frac{\pi}{\delta} \qquad \pi = \frac{\bar{A}_x}{\bar{a}_x}$$

$$\Rightarrow Var(_0L) = \left(1+\frac{\pi}{\delta}\right)^2\left[{}^2\bar{A}_x - (\bar{A}_x)^2\right] \qquad \Rightarrow \frac{\pi}{\delta} = \frac{\bar{A}_x}{\delta \bar{a}_x} = \frac{\bar{A}_x}{1-\bar{A}_x}$$

$$1+\frac{\pi}{\delta} = 1 + \frac{\bar{A}_x}{1-\bar{A}_x} = \frac{1}{1-\bar{A}_x}$$

$$\therefore Var(_0L) = \left(\frac{1}{1-\bar{A}_x}\right)^2\left[{}^2\bar{A}_x - (\bar{A}_x)^2\right] = 0.875$$

5. For a fully discrete 2-year endowment insurance on (x), you are given

(i) The death benefit is 3000 in year 1 and 2000 in year 2

(ii) The maturity benefit is 2000

(iii) Expenses, payable at the beginning of the year are:
Commissions are 50% of the gross premium in the first year and 3% in the second year
Other expenses are 15 in the first year and 2 in the second year

(iv) $i = 0.04$ and $p_x = 0.9$ $\quad q_x = .1$

Determine the annual gross premium using the equivalence principle.

B & E:

0		1		2
15, $.5\pi$	3000	2, $.03\pi$	2000	2000

$$= \begin{cases} EPV(B\&E) = 3000v(.1) + 2000v^2(.9) + .5\pi + 15 + 2v(.9) + .03\pi(.9)v \\ EPV(P) = \pi + \pi v(.9) \end{cases}$$

$$\Rightarrow \pi = \frac{3000v(.1) + 2000v^2(.9) + 15 + 2v(.9)}{1 + v(.9) - .5 - .03v(.9)} = 1470.33$$