MAP 4176 / 5178 Name:____________________

Test 7 Date: March 1, 2017

Show all work for full credit, use correct notation, and clearly mark your answer.
Each problem is worth 10 points.

1. You are given:

   (i) $\pi = \frac{\bar{A}_x}{\bar{a}_x}$

   (ii) $\bar{a}_x = 10$

   (iii) $\delta = 0.06$

   Determine π.

$\bar{a}_x = \frac{1-\bar{A}_x}{\delta} \Rightarrow \bar{A}_x = 1 - \delta \cdot \bar{a}_x$

$\therefore \bar{A}_x = 1 - .06(10) = .4$

$\therefore \pi = \frac{.4}{10} = .04$

2. You are given:

   (i) $d = 0.05$

   (ii) $q_x = 0.02$

   (iii) $A_x = .4$

   Determine the $\ddot{a}_{x+1}$.

$\ddot{a}_x = \frac{1-A_x}{d}$

$\Rightarrow \ddot{a}_x = \frac{1-.4}{.05} = 12$

$\ddot{a}_x = 1 + v p_x \cdot \ddot{a}_{x+1}$

$12 = 1 + (.95)(.98)\ddot{a}_{x+1}$

$\Rightarrow \ddot{a}_{x+1} = 11.815\cdots$

(OR)

$A_x = v q_x + v p_x \cdot A_{x+1}$

$.4 = (.95)(.02) + (.95)(.98) \cdot A_{x+1}$

$\Rightarrow A_{x+1} = .409\ldots$

$\ddot{a}_{x+1} = \frac{1-A_{x+1}}{d} = \frac{1-.409\cdots}{.05}$

$\Rightarrow \ddot{a}_{x+1} = 11.815\cdots$

3. A whole life insurance issued to (40) pays 1000 at the end of the year of death. Using ILT actuarial assumptions, determine the actuarial accumulated value at age 50 of past benefits.

$APV(\text{benefits from age 40 to age 50}) = 1000 \cdot A^1_{40:\overline{10}|}$

$= 1000(A_{40} - {}_{10}E_{40} \cdot A_{50}) \overset{ILT}{=} 27.662\cdots$

$\therefore AAV = \frac{27.662\cdots}{{}_{10}E_{40}} \overset{ILT}{=} 51.544\ldots$

4. For a continuous whole-life annuity issued to (50) with annual payment rate of 1000, you are given:

   (i) Mortality follows the Illustrative Life Table

   (ii) $i = 0.06$

   (iii) Deaths are uniformly distributed between integer ages.

   Determine the variance of the present value random variable for this annuity.

$$Y = 1000\,\bar{Y}_{50} \Rightarrow Var(Y) = 1000^2 \cdot Var(\bar{Y}_{50})$$

$$\bar{Y}_{50} = \frac{1-\bar{Z}_{50}}{\delta} \Rightarrow Var(\bar{Y}_{50}) = \frac{{}^2\bar{A}_{50} - (\bar{A}_{50})^2}{\delta^2}$$

$$\left.\begin{array}{l} \bar{A}_{50} \overset{UDD}{=} \frac{i}{\delta} \cdot A_{50} \overset{ILT}{=} .2564\cdots \\ {}^2\bar{A}_{50} \overset{UDD}{=} \frac{2i+i^2}{2\delta} \cdot {}^2A_{50} \overset{ILT}{=} .1005\cdots \end{array}\right\} \Rightarrow Var(\bar{Y}_{50}) = 10.230\cdots$$

$$\Rightarrow Var(Y) = 10230780.72$$

5. For a whole-life annuity due issued to (30) with monthly payments of 1000, you are given:

   (i) Mortality follows the Illustrative Life Table

   (ii) $i = 0.06$

   (iii) Deaths are uniformly distributed between integer ages.

   Determine the probability that the present value of the payments is less than 100,000.

$$1000\,\ddot{a}_{\overline{n}|m} \overset{M=meir}{=} 100000 \Rightarrow n = 136.4\cdots$$

$\therefore$ PVRV $Y < 100000 \Rightarrow$ there are fewer than 137 payments

$\Rightarrow$ (30) dies within 136 month $(= 11\tfrac{1}{3}$ years$)$

$$\therefore Pr(Y < 100000) = {}_{11\frac{1}{3}}q_{30} = 1 - \frac{l_{41\frac{1}{3}}}{l_{30}}$$

$$l_{41\frac{1}{3}} = \frac{2}{3} \cdot l_{41} + \frac{1}{3} l_{42} \overset{ILT}{=} 9278033$$

$$\therefore Pr(Y < 100000) \overset{ILT}{=} 1 - \frac{9278033}{9501381} = .0235\cdots$$