MAP 4176 / 5178 Name: Key
Test 7 Date: March 7, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. A whole life insurance policy issued to (35) pays 100,000 at the end of the year of death. Determine the actuarial present value of this insurance product using ILT mortality and an annual effective interest rate of 5% for the first year and 6% thereafter.

$$APV = 100000\left(v_{.05}\, q_{35} + v_{.05} \cdot p_{35} \cdot A_{36}\right)$$

$$= 100000\left(\frac{q_{35}}{1.05} + \frac{p_{35}}{1.05} \cdot A_{36}\right) \overset{ILT}{=} 12994.214\ldots$$

2. When Beth was age 30, she purchased a whole life insurance policy with premiums of 1000 payable at the beginning of each year she is alive up to a maximum of 10 years. She is now age 50. Using ILT assumptions, let X denote the accumulated value of her premiums at age 50, and let Y denote the actuarial accumulated value of her premiums at age 50. Determine $X - Y$.

$$X = 1000 \cdot \ddot{s}_{\overline{10}|.06} \cdot (1.06)^{10}$$

$$Y = \frac{1000 \cdot \ddot{a}_{30:\overline{10}|}}{{}_{20}E_{30}} = \frac{1000 \cdot \left(\ddot{a}_{30} - {}_{10}E_{30} \cdot \ddot{a}_{40}\right)}{{}_{20}E_{30}}$$

$$\Rightarrow X - Y = -1350.980\ldots$$

3. For a continuous whole-life insurance of 100,000 issued to (30), you are given:

(i) Mortality follows the Illustrative Life Table

(ii) $\delta = 0.05$

(iii) Deaths are uniformly distributed between integer ages.

Determine the probability that the $\underbrace{\text{present value random variable for this insurance}}_{= Z = 100000 \bar{Z}_{30} = 100000 v^T}$ is less than 20,000.

$$\Pr(Z < 20000) = \Pr(100000 v^T < 20000) = \Pr(v^T < .2)$$

$$= \Pr(e^{-.05T} < .2) = \Pr\left(T > \frac{\ln(.2)}{-.05} = 32.188\ldots\right)$$

$$\therefore \Pr(Z < 20000) = {}_{32.188\ldots}p_{30} = \frac{\ell_{62.188\ldots}}{\ell_{30}} = \frac{(.811\ldots)\ell_{62} + (.188\ldots)\ell_{63}}{\ell_{30}}$$

$$\therefore \Pr(Z < 20000) = 0.8345\ldots$$

4. For a 10-year deferred whole-life insurance issued to (20) with death benefit of 10000 paid at the end of the year of death, you are given:

   (i) Mortality follows the Illustrative Life Table

   (ii) $i = 0.05$

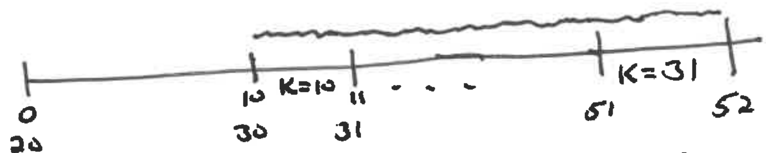
$$\underbrace{Z = 10000\, Z_{20} = 10000\, v^{K+1}}$$

Determine the probability that the present value random variable for this insurance is greater than 2000.

K	Z
≤ 9	0
10	$10000 v^{11} > 2000$
$\vdots$	
31	> 2000
32	< 2000

$\Rightarrow$

$$\therefore \Pr(Z > 2000) = \Pr(10 \le K \le 31)$$



$$\therefore \Pr(Z > 2000) = {}_{10|22}q_{20} = \frac{l_{30} - l_{52}}{l_{20}}$$

$$\Pr(Z > 2000) \overset{ILT}{=} 0.0686\cdots$$

5. For a whole-life annuity due issued to (30) with monthly payments of 1000, you are given:

   (i) Mortality follows the Illustrative Life Table

   (ii) $i = 0.06$

   (iii) Deaths are uniformly distributed between integer ages.

Determine the probability that the present value of the payments is less than 50,000.

Let $n = \#$ of payments. Since $50000 = 1000\, \ddot{a}_{\overline{n}|\, mei'r} \Rightarrow n = 57+$

then $\Pr(Y < 50000) = \Pr(n \le 57)$

$$\therefore \Pr(Y < 50000) = \Pr(T < 57 \text{ months} = 4\tfrac{3}{4} \text{ years}) = \underbrace{\Pr(T < 4.75)}_{= {}_{4.75}q_{30}} = 1 - \frac{l_{34.75}}{l_{30}}$$

$$l_{34.75} = .25 \cdot l_{34} + .75 \cdot l_{35} = 9425135.5$$

$$\therefore \Pr(Y < 50000) = 0.00802\cdots$$