MAP 4176 / 5178 Name:_______________________________

Test 7 Date: February 27, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation.

1. Given $q_{75} = .02$ and d = 10%, determine the standard deviation of the present value random variable for a 2-year temporary annuity due issued to (75) with the first year's payment equal to 1500 and the second year's payment equal to 2000.

Y	Pr
1500	$q_{75} = 0.02$
$1500 + 2000v = 3300$	$p_{75} = 0.98$

$E[Y] = 1500 \cdot 0.02 + 3300 \cdot 0.98 = 3{,}264$

$E[Y^2] = (1500)^2 \cdot 0.02 + (3300)^2 \cdot 0.98 = 10{,}717{,}200$

$$\therefore Var(Y) = E[Y^2] - (E[Y])^2 = 63{,}504 \quad \Rightarrow \quad \sigma_Y = \sqrt{Var(Y)} = 252$$

2. For independent lives (x) and (y), you are given:

(i) Mortality for (x) follows a constant force model with $\mu_x = 0.02$

(ii) Mortality for (y) follows a constant force model with $\mu_y = 0.04$

You are also given $\delta = 0.03$.

Determine the variance of the present value random variable for a continuous annuity that pays an annual rate of 9 per year until the earlier of the death of (x) and (y). (Recall that for independent lives, $\mu_{xy} = \mu_x + \mu_y$)

$$Y = 9 \cdot \bar{Y}_{xy} = 9 \cdot \frac{1-\bar{Z}_{xy}}{\delta} \quad \Rightarrow \quad Var(Y) = 81 \cdot \frac{Var(\bar{Z}_{xy})}{\delta^2} = \frac{81}{0.03^2} \cdot \left({}^2\bar{A}_{xy} - (\bar{A}_{xy})^2 \right)$$

$$\bar{A}_{xy} = \frac{\mu_{xy}}{\mu_{xy}+\delta} = \frac{0.06}{0.09} = \frac{6}{9} = \frac{2}{3}$$

$${}^2\bar{A}_{xy} = \frac{\mu_{xy}}{\mu_{xy}+2\delta} = \frac{0.06}{0.12} = \frac{6}{12} = \frac{1}{2}$$

$$\therefore Var(Y) = 5{,}000$$

3. Use SULT actuarial assumptions and the claims acceleration approach to calculate the variance of the present value random variable for a whole life annuity due issued to (20) with quarterly payments of 250.
   Note that $i = 0.05 \Rightarrow d^{(4)} = 0.04849$.

$$Y = 1000 \cdot \ddot{Y}_{20}^{(4)} = 1000 \cdot \frac{1-Z_{20}^{(4)}}{d^{(4)}}$$

$$\Rightarrow \quad Var(Y) = 1000^2 \cdot \frac{Var\left(Z_{20}^{(4)}\right)}{\left(d^{(4)}\right)^2} = \frac{1000^2}{\left(d^{(4)}\right)^2} \cdot \left({}^2A_{20}^{(4)} - \left(A_{20}^{(4)}\right)^2\right)$$

$$A_{20}^{(4)} = (1+i)^{\frac{3}{8}} \cdot A_{20}$$

$${}^2A_{20}^{(4)} = (1+ {}^2i)^{\frac{3}{8}} \cdot {}^2A_{20} = (1+2i+i^2)^{\frac{3}{8}} \cdot {}^2A_{20} = (1+i)^{\frac{3}{4}} \cdot {}^2A_{20}$$

$$\therefore Var(Y) = 1{,}489{,}940$$

4. Use SULT actuarial assumptions and assume a uniform distribution of deaths between integer ages to determine the variance of the present value random variable for a continuous 10-year temporary annuity of 100 per year issued to (20).

$$Y = 100 \cdot \bar{Y}_{20:\overline{10|}} = 100 \cdot \frac{1-\bar{Z}_{20:\overline{10|}}}{\delta}$$

$$\Rightarrow \quad Var(Y) = 100^2 \cdot \frac{Var\left(\bar{Z}_{20:\overline{10|}}\right)}{\delta^2} = \frac{10000}{\delta^2} \cdot \left({}^2\bar{A}_{20:\overline{10|}} - \left(\bar{A}_{20:\overline{10|}}\right)^2\right)$$

$\bar{A}_{20:\overline{10|}} = \frac{i}{\delta} \cdot A_{20:\overline{10|}}$ For the upper 2 part, we use $A_{20:\overline{10|}} = A_{20} - {}_{10}E_{20} \cdot A_{30} + {}_{10}E_{20}$

$${}^2\bar{A}_{20:\overline{10|}} = \frac{{}^2i}{{}^2\delta} \cdot {}^2A_{20:\overline{10|}} = \frac{2i+i^2}{2\delta} \cdot \left({}^2A_{20} - {}_{10}^{2}E_{20} \cdot {}^2A_{30} + {}_{10}^{2}E_{20}\right)$$

$$\therefore Var(Y) = 739$$

5. For a given annual effective interest rate i, you are given:

   (i) $A_x^{(12)} = 0.7$

   (ii) $\ddot{a}_x^{(12)} = 10$

   Determine i, rounded to 4 decimal places.

$$\ddot{a}_x^{(12)} = \frac{1-A_x^{(12)}}{d^{(12)}} \Rightarrow d^{(12)} = 0.03; \qquad 1+i = \left(1-\frac{d^{(12)}}{12}\right)^{-12} \Rightarrow i = 0.0305$$