MAP 4176 / 5178
Test 8

Name:______________________

Date: March 22, 2017

Show all work for full credit, use correct notation., and clearly mark your answer.

1. For a fully discrete whole life insurance of 50,000 issued to (35) with annual premiums, you are given:

   (i) Mortality follows the Illustrative Life Table

   (ii) $i = 0.06$

   Show that the premium for which the expected value of the loss-at-issue present value random variable equals 0 is 418.

$$E[{}_0L]=0 \Rightarrow EPV(P)=EPV(B) \Rightarrow \pi\cdot\ddot{a}_{35} = 50000\,A_{35}$$

$$\therefore\ \pi = \frac{50000\,A_{35}}{\ddot{a}_{35}} \overset{ILT}{=} 418 \quad (=418.12)$$

2. Using the same actuarial assumptions as in #1, determine the variance of the loss-at-issue present value random variable in #1.

$${}_0L = 50000\,Z_{35} - 418\,\ddot{Y}_{35} = 50000\,Z_{35} - 418\left(\frac{1-Z_{35}}{d}\right)$$

$$\Rightarrow {}_0L = \left(50000+\frac{418}{d}\right)Z_{35} - \frac{418}{d}$$

$$\Rightarrow Var({}_0L) = \left(50000+\frac{418}{.06}(1.06)\right)^2\left[{}^2A_{35}-(A_{35})^2\right] \overset{ILT}{=} 60298656$$

3. For a fully continuous whole life insurance of 1 issued to (x) with annual premium rate, $\pi = 0.04$, you are given:

   (i) $\mu = 0.04$

   (ii) $\delta = 0.06$

   Determine the variance of the loss-at-issue present value random variable.

$${}_0L = \bar{Z}_x - \pi\cdot\bar{Y}_x = \bar{Z}_x - \pi\left(\frac{1-\bar{Z}_x}{\delta}\right) = \left(1+\frac{\pi}{\delta}\right)\bar{Z}_x - \frac{\pi}{\delta}$$

$$\Rightarrow Var({}_0L) = \left(1+\frac{\pi}{\delta}\right)^2\left[{}^2\bar{A}_x - (\bar{A}_x)^2\right]$$

$$\bar{A}_x \overset{CF}{=} \frac{\mu}{\mu+\delta} = \frac{4}{10} = .4 \qquad {}^2\bar{A}_x \overset{CF}{=} \frac{\mu}{\mu+2\delta} = \frac{4}{16} = .25$$

$$\therefore Var({}_0L) = \left(1+\frac{4}{6}\right)^2\left[.25-(.4)^2\right] = 0.25$$

4. For a fully discrete 2-year endowment insurance on (x), you are given

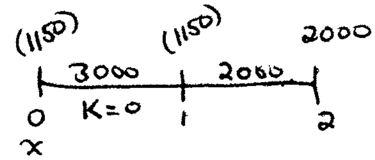
   (i) The death benefit is 3000 in year 1 and 2000 in year 2

   (ii) The maturity benefit is 2000

   (iii) The annual premium is 1150.

   (iv) $p_x = 0.75$

   (v) $d = 0.1$



K	${}_0L$	Pr
0	$3000v-\pi = 1550$	.25
≥ 1	$2000v^2-\pi-\pi v = -565$	.75

Determine the variance of the loss-at-issue present value random variable for this insurance.

$({}_0L)^2$	${}_0L$	Pr
2402500	1550	.25
319225	-565	.75

$$Var({}_0L) = E[({}_0L)^2] - (E[{}_0L])^2$$

$$\Rightarrow Var({}_0L) = 838730$$

5. For a fully discrete 20-year endowment insurance of 1000 issued to (40), you are given:

   (i) The death benefit is paid at the end of the month of death.

   (ii) A premium of 3 is paid at the beginning of each month.

   (iii) There is a uniform distribution of deaths between integer ages.

   (iv) ${}^2A^{(12)}_{40:\overline{20|}} = 0.12097$

Using ILT actuarial assumptions, determine the variance of the loss-at-issue present value random variable for this insurance.

$${}_0L = 1000\,Z^{(12)}_{40:\overline{20|}} - 3(12)\cdot \ddot{Y}^{(12)}_{40:\overline{20|}} = 1000\,Z^{(12)}_{40:\overline{20|}} - 36\cdot\left(\frac{1-Z^{(12)}_{40:\overline{20|}}}{d^{(12)}}\right)$$

$$\Rightarrow {}_0L = \left(1000 + \frac{36}{d^{(12)}}\right) Z^{(12)}_{40:\overline{20|}} - \frac{36}{d^{(12)}} \Rightarrow Var({}_0L) = \left(1000+\frac{36}{d^{(12)}}\right)^2\left[{}^2A^{(12)}_{40:\overline{20|}} - \left(A^{(12)}_{40:\overline{20|}}\right)^2\right]$$

$d^{(12)} \overset{ILT}{=} .05813$ Need $A^{(12)}_{40:\overline{20|}}$

Directly: $A^{(12)}_{40:\overline{20|}} = A^{(12)\,1}_{40:\overline{20|}} + {}_{20}E_{40}$

$\overset{UDD}{=} \frac{i}{i^{(12)}}\cdot A^{1}_{40:\overline{20|}} + {}_{20}E_{40}$

$= \frac{i}{i^{(12)}}\left[A_{40} - {}_{20}E_{40}\cdot A_{60}\right] + {}_{20}E_{40} \overset{ILT}{=} .3359\ldots$

$\therefore Var({}_0L) = 21342$

Indirectly:

$\ddot{a}^{(12)}_{40:\overline{20|}} = \ddot{a}^{(12)}_{40} - {}_{20}E_{40}\cdot\ddot{a}^{(12)}_{60}$

$\ddot{a}^{(12)}_{x} \overset{UDD}{=} \alpha(12)\,\ddot{a}_x - \beta(12)$

$\therefore \ddot{a}^{(12)}_{40:\overline{20|}} \overset{ILT}{=} 11.4247\ldots = \frac{1-A^{(12)}_{40:\overline{20|}}}{d^{(12)}}$

$\therefore Var({}_0L) = 21379$