MAP 4176 / 5178 Name: KEY

Test 8 Date: March 21, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. For a fully continuous whole life insurance of 1000 issued to (x) with annual premium rate of 35, use $CF(\mu = 0.02, \delta = 0.03)$ assumptions to determine the variance of the loss-at-issue present value random variable.

$$L = 1000\bar{Z}_x - 35\cdot\bar{Y}_x = \left(1000 + \tfrac{35}{\delta}\right)\cdot\bar{Z}_x - \tfrac{35}{\delta} \quad \text{since } \bar{Y}_x = \frac{1-\bar{Z}_x}{\delta}$$

$$\therefore Var(L) = \left(1000 + \tfrac{35}{.03}\right)^2\cdot\left[{}^2\bar{A}_x - (\bar{A}_x)^2\right]$$

$$\bar{A}_x \overset{CF}{=} \frac{\mu}{\mu+\delta} = \frac{2}{5} \qquad {}^2\bar{A}_x \overset{CF}{=} \frac{\mu}{\mu+2\delta} = \frac{2}{8}$$

$$\therefore Var(L) = 422{,}500$$

2. Given a whole life insurance of 10000, issued to a 52 year old, with the death benefit payable at the end of the year of death, and a single premium of 2800, determine the variance of the loss-at-issue random variable using ILT actuarial assumptions.

$$L = 10000\,Z_{52} - 2800$$

$$\therefore Var(L) = 10000^2\cdot\left({}^2A_{52} - (A_{52})^2\right) \overset{ILT}{=} 3{,}474{,}975$$

3. For a 3-year semi-continuous endowment insurance issued to (40), you are given:

i) the death benefit is 10000 and is payable at the moment of death
ii) the amount of the pure endowment is 10000
iii) annual premiums are 3100
iv) $i = .05$

At what time would the 40-year old have to die in order for the loss-at-issue random variable to be equal to 0.

$$L = 10000\,\bar{Z}_{40:\overline{3}|} - 3100\,\ddot{Y}_{40:\overline{3}|}$$

If $T < 2$, there are only 2 premiums paid, and $L > 0$

If $T \geq 3$, then $L = 10000v^3 - 3100\,\ddot{a}_{\overline{3}|} = -225.796\cdots < 0$

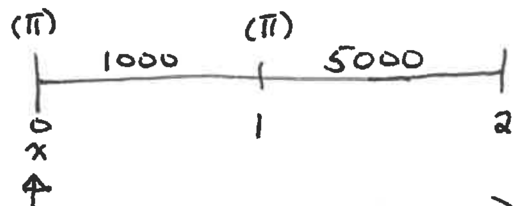
$$\therefore 2 \leq T < 3 \Rightarrow L = 10000\,v_{.05}^T - 3100\,\ddot{a}_{\overline{3}|.05} = 0$$

$$\Rightarrow T = 2.471\cdots$$

4. For a fully discrete 2-year term insurance issued to (x), you are given:

   i) the death benefit is 1000 in the first year and 5000 in the second year
   ii) $d = .1$
   iii) $p_x = 0.9$ and $p_{x+1} = 0.8$

   Determine the annual premium which makes the expected loss-at-issue equal 0.



$$E[L] = 0 \Rightarrow APV(\text{Ben.}) = APV(\text{Prem.})$$

$$\therefore \ 1000\, v\, q_x + 5000\, v^2 \cdot {}_{1|}q_x = \pi + \pi\, v\, p_x \qquad v = .9$$

$$\therefore \ \pi = \frac{1000(.9)(.1) + 5000(.9)^2 \cdot (.9)(.2)}{1 + .9(.9)} = 452.486\cdots$$

5. A 20-year deferred whole life annuity due issued to (40) pays 60,000 at the beginning of each year. Premiums of 21,000 are paid at the beginning of each year during the deferred period. Determine the expected loss-at-issue using ILT assumptions.

$$L = 60000 \cdot {}_{20|}\ddot{Y}_{40} - 21000 \cdot \ddot{Y}_{40:\overline{20}|}$$

$$\Rightarrow E[L] = 60000 \cdot {}_{20|}\ddot{a}_{40} - 21000\, \ddot{a}_{40:\overline{20}|}$$

$${}_{20|}\ddot{a}_{40} = {}_{20}E_{40} \cdot \ddot{a}_{60} \overset{ILT}{=} 3.055\cdots$$

$$\ddot{a}_{40:\overline{20}|} = \ddot{a}_{40} - {}_{20}E_{40} \cdot \ddot{a}_{60} \overset{ILT}{=} 11.761\cdots$$

$$\therefore E[L] = -63,661.203\cdots$$