MAP 4176 / 5178 Name:_______________________________

Test 8 Date: March 6, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation.

1. You are given:

   (i) $d = 0.10$

   (ii) $q_{75} = 0.02$

   (iii) $A_{76} = 0.52$

   Determine $A_{75}$.

$$A_{75} = v \cdot q_{75} + v \cdot p_{75} \cdot A_{76} = 0.90(0.02) + 0.90(0.98)(0.52) = 0.47664$$

2. A company issues $n$ independent identical continuous whole life insurances to 35-year olds with benefit of 10,000. Using $CF(\mu = 0.03, \delta = 0.03)$ actuarial assumptions and the normal approximation, the probability that the total present value of all benefits paid exceeds 2,500,000 is 0.5. Determine $n$.
   (You don't need the standard normal distribution table to complete this question.)

   Let $T$ denote the random variable representing the total present value of all benefits paid. Then $T = \sum_0^n 10000 \cdot \bar{Z}_{35}$.

   $\Pr(T > 2{,}500{,}000) = 0.5 \quad \Rightarrow \quad \Pr\left(SND > \frac{2{,}500{,}000 - E[T]}{\sqrt{Var(T)}}\right) = 0.5$

   $\therefore \frac{2{,}500{,}000 - E[T]}{\sqrt{Var(T)}} = 0$ and so $E[T] = 2{,}500{,}000$.

   $E[T] = n \cdot 10000 \cdot \bar{A}_{35} = n \cdot 10000 \cdot \frac{\mu}{\mu + \delta} = 5000n$

$$\therefore n = 500$$

3. Determine the actuarial accumulated value at age 40 of a discrete 10-year term insurance of 10,000 issued to (20), using SULT actuarial assumptions.

$$AAV = \frac{10000 \cdot \left(A_{20:\overline{10|}} - {}_{10}E_{20}\right)}{{}_{20}E_{20}} = 55.82$$

4. A whole life annuity issued to (40) pays 500 at the end of each year. Using $i = 0.06$ and $DML(\omega = 90)$ actuarial assumptions, determine the probability that the sum of the payments made is greater than or equal to 10,000.

Let $\Sigma$ denote the sum of the payments made. Note that $\Sigma$ is just the sum of the payments, not the present value of the payments made. We seek $\Pr(\Sigma \geq 10000)$.

Since $\Sigma = 500n$, where $n$ is the number of payments made, then
$\Pr(\Sigma \geq 10000) = \Pr(n \geq 20)$.

Since the 20th payment occurs when (40) reaches age 40, $\Pr(n \geq 20) = \Pr(K \geq 20)$.

$$\therefore \Pr(\Sigma \geq 10000) = \Pr(K \geq 20) = {}_{20}p_{40} = \frac{90 - 40 - 20}{90 - 40} = 0.6$$

5. A 10-year deferred whole life insurance issued to $(x)$ pays 100,000 at the end of the quarter of death. Using a constant force of mortality, $\mu = 0.03$, and $i = 0.05$, determine the probability that the present value of the benefit is less than 32,000.

| $K^{(4)}$ | $Z$ | Pr |
|---|---|---|
| $< 10.00$ | 0 | ${}_{10}q_x$ |
| 10.00 | $100000v^{10.25} = 60647 + (> 32000)$ | |
| $\vdots$ | $\vdots$ | $\vdots$ |
| 23.00 | $100000v^{23.25} = 32162 + (> 32000)$ | |
| 23.25 | $100000v^{23.50} = 31772 + (< 32000)$ | |
| $\vdots$ | $\vdots$ | |

$\therefore \Pr(Z < 32000) = \Pr\left(K^{(4)} < 10\right) + \Pr\left(K^{(4)} \geq 23.25\right)$

$\Pr\left(K^{(4)} < 10\right) = {}_{10}q_x = 1 - e^{-10\mu}$ and $\Pr\left(K^{(4)} \geq 23.25\right) = {}_{23.25}p_x = e^{-23.25\mu}$

$$\therefore \Pr(Z < 32000) = 0.757\cdots$$