Each problem is worth 10 points. Show all work for full credit, and use correct notation.

1. For a fully discrete whole life insurance of 1000 issued to (20) with annual premiums of 3, use SULT actuarial assumptions to determine the variance of the loss-at-issue present value random variable.

\[ 0L = 1000 \cdot Z_{20} - 3 \cdot Y_{20} = 1000 \cdot Z_{20} - 3 \cdot \frac{1 - Z_{20}}{d} = \left(1000 + \frac{3}{d}\right) \cdot Z_{20} - \frac{3}{d} \]

\[ \Rightarrow \quad Var(0L) = \left(1000 + \frac{3}{d}\right)^2 \cdot Var(Z_{20}) = \left(1000 + \frac{3}{d}\right)^2 \cdot \left(2A_{20} - (A_{20})^2\right) \]

\[ \therefore Var(0L) = 3816 \]

2. For a fully discrete whole life insurance of 1000 issued to (20) with annual premiums determined by the equivalence principle, use SULT actuarial assumptions to determine the variance of the loss-at-issue present value random variable.

Let \( \pi \) denote the annual premium. Then \( \pi = \frac{1000A_{20}}{d_{20}} = \frac{1000A_{20} \cdot d}{1 - A_{20}} \)

\[ 0L = 1000 \cdot Z_{20} - \pi \cdot Y_{20} = 1000 \cdot Z_{20} - \pi \cdot \frac{1 - Z_{20}}{d} = \left(1000 + \frac{\pi}{d}\right) \cdot Z_{20} - \frac{\pi}{d} \]

\[ \Rightarrow \quad Var(0L) = \left(1000 + \frac{\pi}{d}\right)^2 \cdot Var(Z_{20}) = \left(1000 + \frac{\pi}{d}\right)^2 \cdot \left(2A_{20} - (A_{20})^2\right) \]

From the last equation in the first line of the solution \( \frac{\pi}{d} = \frac{1000A_{20}}{1 - A_{20}} \). So,

\[ 1000 + \frac{\pi}{d} = 1000 \cdot \left(1 + \frac{A_{20}}{1 - A_{20}}\right) = 1000 \cdot \left(\frac{1}{1 - A_{20}}\right) \]

\[ \therefore Var(0L) = 1000^2 \cdot \frac{2A_{20} - (A_{20})^2}{(1 - A_{20})^2} = 3736 \]
3. For an insurance issued to independent lives \((x)\) and \((y)\), a benefit of 10,000 is paid at the moment of the second death. Premiums are paid continuously at an annual rate of \(\pi\) until the first death. Using \(CF(\mu_x = 0.01, \mu_y = 0.02, \delta = 0.03)\) actuarial assumptions and the equivalence principle, determine \(\pi\).

\[
\pi = \frac{10000 \cdot \delta_{xy}}{\delta_{xy}} = 10000 \cdot \frac{A_x + A_y - \Delta_{xy}}{\delta_{xy}}
\]

\[
A_x = \frac{\mu_x}{\mu_x + \delta} = \frac{1}{4}, \quad A_y = \frac{\mu_y}{\mu_y + \delta} = \frac{2}{5}, \quad A_{xy} = \frac{\mu_{xy}}{\mu_{xy} + \delta} = \frac{\mu_x + \mu_y}{\mu_x + \mu_y + \delta} = \frac{3}{6} = \frac{1}{2}
\]

\[
\delta_{xy} = \frac{1}{\mu_{xy} + \delta} = \frac{1}{\mu_x + \mu_y + \delta} = \frac{1}{0.06}
\]

\[
\therefore \pi = 90
\]

4. For a fully discrete 2-year term insurance issued to \((x)\), you are given:

i) the death benefit is 3000 in the first year and 5000 in the second year
ii) \(d = 0.05\)
iii) \(q_x = 0.05\) and \(1|q_x = 0.04\)

Determine the net annual premium.

\[
APV(Ben) = 3000 \cdot v \cdot q_x + 5000 \cdot v^2 \cdot 1|q_x = 323
\]

\[
APV(Prem) = \pi + \pi \cdot v \cdot p_x = 1.9025
\]

\[
\therefore \pi = \frac{323}{1.9025} = 170
\]

5. For a fully discrete whole life insurance of 10,000 issued to \((x)\) with annual premiums of 75, using \(i = 0.05\), determine the minimum value of the curtate future lifetime random variable, \(K\), such that the value of the loss-at-issue present value random variable is negative.

\[
0L = 10000 \cdot Z_x - 75 \cdot \bar{Y}_x = 10000 \cdot v^{K+1} - 75 \cdot \delta_{K+1}^{11}
\]

Using TVM or guess-and-check, note that \(10000 \cdot v^{40} - 75 \cdot \delta_{40}^{11} > 0\) but \(10000 \cdot v^{41} - 75 \cdot \delta_{41}^{11} < 0\).

So if \(K = 39\) (or less), then \(0L > 0\), but if \(K = 40\) (or greater), then \(0L < 0\). Therefore the minimum value of \(K\) such that \(0L < 0\) is \(K = 40\).