MAP 4176 / 5178 Name:______________________________

Test 9 Date: March 13, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation.

1. For a fully discrete whole life insurance of 1000 issued to (20) with annual premiums of 3, use SULT actuarial assumptions to determine the variance of the loss-at-issue present value random variable.

$${}_0L = 1000 \cdot Z_{20} - 3 \cdot \ddot{Y}_{20} = 1000 \cdot Z_{20} - 3 \cdot \frac{1-Z_{20}}{d} = \left(1000 + \frac{3}{d}\right) \cdot Z_{20} - \frac{3}{d}$$

$$\Rightarrow \quad Var\left({}_0L\right) = \left(1000 + \frac{3}{d}\right)^2 \cdot Var(Z_{20}) = \left(1000 + \frac{3}{d}\right)^2 \cdot \left({}^2A_{20} - (A_{20})^2\right)$$

$$\therefore Var\left({}_0L\right) = 3816$$

2. For a fully discrete whole life insurance of 1000 issued to (20) with annual premiums determined by the equivalence principle, use SULT actuarial assumptions to determine the variance of the loss-at-issue present value random variable.

Let π denote the annual premium. Then $\pi = \frac{1000A_{20}}{\ddot{a}_{20}} = \frac{1000A_{20} \cdot d}{1-A_{20}}$

$${}_0L = 1000 \cdot Z_{20} - \pi \cdot \ddot{Y}_{20} = 1000 \cdot Z_{20} - \pi \cdot \frac{1-Z_{20}}{d} = \left(1000 + \frac{\pi}{d}\right) \cdot Z_{20} - \frac{\pi}{d}$$

$$\Rightarrow \quad Var\left({}_0L\right) = \left(1000 + \frac{\pi}{d}\right)^2 \cdot Var(Z_{20}) = \left(1000 + \frac{\pi}{d}\right)^2 \cdot \left({}^2A_{20} - (A_{20})^2\right)$$

From the last equation in the first line of the solution $\frac{\pi}{d} = \frac{1000A_{20}}{1-A_{20}}$. So,
$1000 + \frac{\pi}{d} = 1000 \cdot \left(1 + \frac{A_{20}}{1-A_{20}}\right) = 1000 \cdot \left(\frac{1}{1-A_{20}}\right)$

$$\therefore Var\left({}_0L\right) = 1000^2 \cdot \frac{{}^2A_{20} - (A_{20})^2}{(1 - A_{20})^2} = 3736$$

3. For an insurance issued to independent lives (x) and (y), a benefit of 10,000 is paid at the moment of the second death. Premiums are paid continuously at an annual rate of π until the first death. Using $CF(\mu_x = 0.01, \mu_y = 0.02, \delta = 0.03)$ actuarial assumptions and the equivalence principle, determine π.

$\pi = \frac{10000 \cdot \bar{A}_{\overline{xy}}}{\bar{a}_{xy}} = 10000 \cdot \frac{\bar{A}_x + \bar{A}_y - \bar{A}_{xy}}{\bar{a}_{xy}}$

$\bar{A}_x = \frac{\mu_x}{\mu_x + \delta} = \frac{1}{4}$, $\quad \bar{A}_y = \frac{\mu_y}{\mu_y + \delta} = \frac{2}{5}$, $\quad \bar{A}_{xy} = \frac{\mu_{xy}}{\mu_{xy} + \delta} = \frac{\mu_x + \mu_y}{\mu_x + \mu_y + \delta} = \frac{3}{6} = \frac{1}{2}$

$\bar{a}_{xy} = \frac{1}{\mu_{xy} + \delta} = \frac{1}{\mu_x + \mu_y + \delta} = \frac{1}{0.06}$

$$\therefore \pi = 90$$

4. For a fully discrete 2-year term insurance issued to (x), you are given:

i) the death benefit is 3000 in the first year and 5000 in the second year
ii) $d = 0.05$
iii) $q_x = 0.05$ and ${}_{1|}q_x = 0.04$

Determine the net annual premium.

$APV(Ben) = 3000 \cdot v \cdot q_x + 5000 \cdot v^2 \cdot {}_{1|}q_x = 323$

$APV(Prem) = \pi + \pi \cdot v \cdot p_x = 1.9025$

$$\therefore \pi = \frac{323}{1.9025} = 170$$

5. For a fully discrete whole life insurance of 10,000 issued to (x) with annual premiums of 75, using $i = 0.05$, determine the minimum value of the curtate future lifetime random variable, K, such that the value of the loss-at-issue present value random variable is negative.

${}_0L = 10000 \cdot Z_x - 75 \cdot \ddot{Y}_x = 10000 \cdot v^{K+1} - 75 \cdot \ddot{a}_{\overline{K+1|}}$

Using TVM or guess-and-check, note that $10000 \cdot v^{40} - 75 \cdot \ddot{a}_{\overline{40|}} > 0$ but $10000 \cdot v^{41} - 75 \cdot \ddot{a}_{\overline{41|}} < 0$.

So if $K = 39$ (or less), then ${}_0L > 0$, but if $K = 40$ (or greater), then ${}_0L < 0$. Therefore the minimum value of K such that ${}_0L < 0$ is $K = 40$.