# Solutions to MLCM 3S7 Exercises

1) See Video Solution

$P = .9772$

2) $S = \sum_{1}^{10000} ({}_0L)_i \qquad {}_0L = 100000\, Z_{60} - \pi\, \ddot{Y}_{60}$

We seek $\pi$ such that $Pr(S > 0) = .01$

$E[S] = 10000\, E[{}_0L]$

$E[{}_0L] = 100000\, A_{60} - \pi\, \ddot{a}_{60} \overset{ILT}{=} 36913 - 11.1454\pi$

$Var(S) = 10000\, Var({}_0L) \qquad \ddot{Y}_{60} = \frac{1 - Z_{60}}{d}$

${}_0L = \left(100000 + \frac{\pi}{d}\right) Z_{60} - \frac{\pi}{d}$

$Var({}_0L) = \left(100000 + \frac{\pi}{d}\right)^2 \left[{}^2A_{60} - (A_{60})^2\right]$

$\therefore Var(S) = 10000 \left(100000 + \frac{\pi}{d}\right)^2 \left[{}^2A_{60} - (A_{60})^2\right]$

$\Rightarrow \sqrt{Var(S)} = 100\left(100000 + \frac{\pi}{d}\right)\sqrt{{}^2A_{60} - (A_{60})^2} \qquad d = \frac{.06}{1.06}$

$\overset{ILT}{=} 2028621.283 + 358.38976\pi$

$\therefore Pr(S > 0) = Pr\left(SND > \frac{-E[S]}{\sqrt{Var(S)}} = \underbrace{\frac{(11.1454\pi - 36913)(10000)}{2028621.283 + 358.38976\pi}}_{= 99^{th} \text{ percentile of SND}}\right) = .01$

standard normal distribution $\qquad \Rightarrow$

$\therefore \frac{10000(11.1454\pi - 36913)}{2028621.283 + 358.38976\pi} = 2.326 \Rightarrow \pi = 3379.56$

3) $${}_0L = 5000\bar{Z}_x - \pi \cdot \bar{Y}_x \qquad \bar{Y}_x = \frac{1-\bar{Z}_x}{\delta}$$

$$= \left(5000 + \frac{\pi}{\delta}\right)\bar{Z}_x - \frac{\pi}{\delta} \overset{\delta = .04}{=\!=} (5000 + 25\pi)\bar{Z}_x - 25\pi$$

$$\therefore \quad .95 = Pr({}_0L < 0) = Pr\left(\bar{Z}_x < \frac{25\pi}{5000+25\pi}\right)$$

$$\bar{Z}_x = e^{-\delta T} = e^{-.04T}$$

$$\Rightarrow .95 = Pr\left(\bar{Z}_x < \frac{25\pi}{5000+25\pi}\right)$$

$$= Pr\left(T > \frac{\ln\left(\frac{25\pi}{5000+25\pi}\right)}{-.04}\right)$$

$$= Pr\left(T > \underbrace{-25\ln\left(\frac{25\pi}{5000+25\pi}\right)}_{=: t}\right)$$

$$= {}_tP_x = e^{-\mu t}$$

$$= e^{-.02\left[-25\ln\left(\frac{25\pi}{5000+25\pi}\right)\right]}$$

$$= e^{.5\ln\left(\frac{25\pi}{5000+25\pi}\right)} = \left(\frac{25\pi}{5000+25\pi}\right)^{.5}$$

$$\therefore \quad (.95)^2 = \frac{25\pi}{5000+25\pi} \Rightarrow \pi = 1851.28$$

4) $\quad {}_0L = 5000\, Z_{35} - \pi \cdot \ddot{Y}_{35} = 5000\, v^{K+1} - \pi\, \ddot{a}_{\overline{K+1}|}$

| | | ${}_0L$ | $Pr$ | $K$ | |
|---|---|---|---|---|---|
| ${}_0L > 0$ | $(+)$ | $5000v - \pi$ | $q_{35}$ | $0$ | $\Sigma = {}_n q_{35}$ |
| | $(+)$ | $5000v^2 - \pi \ddot{a}_{\overline{2}|}$ | ${}_{1|}q_{35}$ | $1$ | |
| | | $\vdots$ | $\vdots$ | $\vdots$ | |
| | $(+)$ | $5000v^n - \pi \ddot{a}_{\overline{n}|}$ | ${}_{(n-1)|}q_{35}$ | $n-1$ | |
| ${}_0L < 0$ | $(-)$ | $5000v^{n+1} - \pi \ddot{a}_{\overline{n+1}|}$ | ${}_{n|}q_x$ | $n$ | $\Sigma = {}_n P_{35}$ |
| | | $\vdots$ | $\vdots$ | $\vdots$ | |

$$Pr({}_0L < 0) = {}_nP_{35} > .95 \Rightarrow \frac{\ell_{35+n}}{\ell_{35}} > .95$$

$$\Rightarrow \ell_{35+n} > .95(\ell_{35}) \overset{ILT}{=} 8949624$$

Since $\ell_{50} = 8950901$

but $\ell_{51} = 8897913$, choose $n = 15$

$\therefore$ we want $\pi$ such that

$$5000\, v_{.06}^{16} - \pi\, \ddot{a}_{\overline{16}|.06} < 0$$

$$\Rightarrow \pi > \frac{5000\, v^{16}}{\ddot{a}_{\overline{16}|}} \Rightarrow \pi > 183.74$$