Module 4 Section 7 Exercises:

1. For a fully continuous whole life insurance of 100 issued to (x), you are given:

(i) $\mu_{x+t} = 0.02t$ and $\delta = 0.05$
(ii) Premiums are determined using the equivalence principle.
(iii) The annual premium rate for the first year is 5.

(a) Determine the net premium reserve at time 0

(b) Using Thiel's Differential Equation (TDE), determine the value of the derivative of the reserve, evaluated at time 0, using your answer from part (a)

(c) Use Euler's Forward Equation with $h = 0.1$, along with your answers from parts (a) and (b), to determine an approximate value of ${}_{0.1}V$

(d) Using TDE, determine an approximation for the derivative of the reserve, evaluated at time 0.1, using the approximate value of ${}_{0.1}V$ found in part (c)

(e) Use Euler's Forward Equation with $h = 0.1$, along with your answers from parts (c) and (d), to determine an approximate value of ${}_{0.2}V$

2. This is the same set-up as Exercise 1. It is repeated here for convenience.

For a fully continuous whole life insurance of 100 issued to (x), you are given:

(i) $\mu_{x+t} = 0.02t$ and $\delta = 0.05$
(ii) Premiums are determined using the equivalence principle.
(iii) The annual premium rate for the first year is 5.

(a) Determine the net premium reserve at time 0

(b) Using TDE, determine an expression for the derivative of the reserve, evaluated at time 0.1, where the expression depends on the unknown ${}_{0.1}V$

(c) Use Euler's Backward Equation with $h = -0.1$, along with your answers from parts (a) and (b), to determine an approximate value of ${}_{0.1}V$

(d) Using TDE, determine an expression for the derivative of the reserve, evaluated at time 0.2, where the expression depends on the unknown ${}_{0.2}V$

(e) Use Euler's Backward Equation with $h = -0.1$, along with your answers from parts (c) and (d), to determine an approximate value of ${}_{0.2}V$

3. For a fully continuous 5-year endowment insurance issued to (40), you are given:

(i) The death benefit at time t is $S(t) = 100t$
(ii) The pure endowment is 500
(iii) $\mu_{x+t} = 0.001 \cdot (1.1)^{x+t}$ and $\delta_t = 0.01t$
(iv) The annual gross premium rate at time t is $\pi_t = 2 + t$
(v) Non-settlement expenses are paid continuously at a rate of $e_t = 1 + 0.5t$
(vi) The settlement expense, paid upon death at time t, is $10t$
(vii) There is no settlement expense for the pure endowment

(a) Determine the net premium reserve at time 5

(b) Using TDE, determine the value of the derivative of the reserve, evaluated at time 5, using your answer from part (a)

(c) Use Euler's Backward Equation with $h = -0.5$, along with your answers from parts (a) and (b), to determine an approximate value of ${}_{4.5}V$

(d) Using TDE, determine an approximation for the derivative of the reserve, evaluated at time 4.5, using the approximate value of ${}_{4.5}V$ found in part (c)

(e) Use Euler's Backward Equation with $h = -0.5$, along with your answers from parts (c) and (d), to determine an approximate value of ${}_{4}V$

4. This is the same set-up as Exercise 3. It is repeated here for convenience.

For a fully continuous 5-year endowment insurance issued to (40), you are given:

(i) The death benefit at time t is $S(t) = 100t$
(ii) The pure endowment is 500
(iii) $\mu_{x+t} = 0.001 \cdot (1.1)^{x+t}$ and $\delta_t = 0.01t$
(iv) The annual gross premium rate at time t is $\pi_t = 2 + t$
(v) Non-settlement expenses are paid continuously at a rate of $e_t = 1 + 0.5t$
(vi) The settlement expense, paid upon death at time t, is $10t$
(vii) There is no settlement expense for the pure endowment

(a) Determine the net premium reserve at time 5

(b) Using TDE, determine an expression for the derivative of the reserve, evaluated at time 4.5, where the expression depends on the unknown ${}_{4.5}V$

(c) Use Euler's Forward Equation with $h = 0.5$, along with your answers from parts (a) and (b), to determine an approximate value of ${}_{4.5}V$

(d) Using TDE, determine an expression for the derivative of the reserve, evaluated at time 4.0, where the expression depends on the unknown ${}_{4.0}V$

(e) Use Euler's Forward Equation with $h = 0.5$, along with your answers from parts (c) and (d), to determine an approximate value of ${}_{4.0}V$