Solutions to MLCM5S2 Exercises

1) $AS_0 = 0 = 1000\, v\, q_x + \underbrace{.6(100) - 100}_{=-40} + AS_1 \cdot v\, p_x$

$$\Rightarrow AS_1 = \frac{40(1+i) - 1000\, q_x}{p_x} \qquad q_x = .03 \quad p_x = .97$$

$$\therefore AS_1 = \frac{40(1.1) - 1000(.03)}{.97} \doteq 14.43$$

2) (a) $EDB_1 = 1000\, q_x = 1000\,(.03) = 30$

(b) $Pr_1 = ({}_0V + \pi - e)(1+i) - 1000\, q_x - {}_1V \cdot p_x$

Since reserves are FPT reserves,

$${}_0V = {}_0V^{FPT} = 0$$

and $${}_1V = {}_1V^{FPT} = 0$$

$$\therefore Pr_1 = (100 - .6(100))(1.1) - 1000(.03)$$

$$= 14$$

3) We still have ${}_0V = {}_0V^{FPT} = 0$ and ${}_1V = {}_1V^{FPT} = 0$.

$\therefore Pr_1 = (100 - e)(1+i) - 1000 q_x$ per policy sold

$\therefore$ per policy sold,

$e^{expected} = .6(100) = 60$

$e^{actual} = \dfrac{25000}{500} = 50$

$i^{expected} = .1$

$i^{actual} = .08$

$q_x^{expected} = .03$

$q_x^{actual} = \dfrac{15}{500} = .03$

(a) $G^{(T)} = \left[Pr_1\,(A-e, i, m)\right] - \left[Pr_1\,(E-e, i, m)\right]$

$= \left[(100-50)(1.08) - 1000(.03)\right] - \left[(100-60)(1.1) - 1000(.03)\right]$

$= 50(1.08) - 40(1.1) = 10$ per policy sold

$\therefore$ the total gain on all policies for year 1 is $500(10) = 5000$

3) (b) $G^i = Pr\begin{pmatrix} A-i \\ E-e,m \end{pmatrix} - Pr\begin{pmatrix} E-i \\ E-e,m \end{pmatrix}$

$$= \left[(100-60)(1.08) - 1000(.03)\right] - \left[(100-60)(1.1) - 1000(.03)\right]$$

$= -0.8$ per policy sold

$\therefore$ the total gain on all policies is $500(-0.8) = -400$

(c) $G^m = Pr\begin{pmatrix} A-m \\ E-i,e \end{pmatrix} - Pr\begin{pmatrix} E-m \\ E-i,e \end{pmatrix}$

$$= \left[(100-60)(1.1) - 1000(.03)\right] - \left[(100-60)(1.1) - 1000(.03)\right]$$

$= 0$ as expected since actual mortality equals expected mortality

$\therefore$ the total gain on all policies is $500(0) = 0$.

(d) $G^{i,m} = Pr\begin{pmatrix} A-i,m \\ E-e \end{pmatrix} - Pr\begin{pmatrix} A-i \\ E-m,e \end{pmatrix}$

$$= \left[(100-60)(1.08) - 1000(.03)\right] - \left[(100-60)(1.08) - 1000(.03)\right]$$

$= 0$ as expected for the same reason as in part (c)

$\therefore$ the total gain on all policies in this case is $500(0) = 0$.

3(e) $\quad G^{e \text{ after } \{m,i\}} = Pr\begin{pmatrix} A-e \\ A-m,i \end{pmatrix} - Pr\begin{pmatrix} E-e \\ A-m,i \end{pmatrix}$

$= \left[(100-50)(1.08) - 1000(.03)\right] - \left[(100-60)(1.08) - 1000(.03)\right]$

$= +10.8$ per policy sold

$\therefore$ the total gain on all policies in this case is $500(+10.8) = 5400$

Note: $\quad G^{(\tau)} = \underbrace{G^{i}}_{\substack{\text{from part (b)} \\ = -400}} + \underbrace{G^{i,m}}_{\substack{\text{from part (d)} \\ = 0}} + \underbrace{G^{e \text{ after } \{m,i\}}}_{\substack{\text{from part (e)} \\ = 5400}}$

$G^{(\tau)}$ from part (a) $= 5000$

4) $AS_{10} = (S+E)v\,q^{(d)}_{x+10} + (W-SC)v\,q^{(w)}_{x+10} + e - \Pi + AS_{11} \cdot v\, p^{(\tau)}_{x+10}$

$= (10000+100)(.9)(.02) + (1500-50)(.9)(.1) + .05(300)$

$\quad -300 + 230(.9)(1-.02-.1)$

$\Longrightarrow AS_{10} = 209.46$

5) Since reserves are "gross premium reserves" then $Pr_{10} = 0$. Everything else is just noise.

6) (a) $$EDB_{11} = (S+E)\cdot q^{(d)}_{x+10} = (10000+100)(.02) = 202$$

(b) $\cancel{ECV_{11} = (1500 - 6}$

$$ECV_{11} = (W - SC)\cdot q^{(w)}_{x+10} = (1500-50)(.1) = 145$$

(c) $$Pr_{11} = ({}_{10}V + \pi - e)(1+i) - \overbrace{(S+E)\cdot q^{(d)}_{x+10}}^{=EDB_{11}} - \overbrace{(W-SC)\cdot q^{(w)}_{x+10}}^{=ECV_{11}} - {}_{11}V\cdot p^{(\tau)}_{x+10}$$

$$\therefore Pr_{11} = (1500 + 300 - .05(300))(1.1) - 202 - 145 - 1750(1 - .02 - .1)$$

$$= 76.5$$

7) $Pr_{11} = ({}_{10}V + \pi - e)(1+i) - (S+E)\, q_{x+10}^{(d)} - (W - SC)\, q_{x+10}^{(w)} - {}_{11}V \cdot p_{x+10}^{(\tau)}$

(a) $$G^{(\tau)} = Pr_{11}(A-e,i,m,w) - \underbrace{Pr_{11}(E-e,i,m,w)}_{=76.5 \text{ from } 6(c)}$$

$Pr_{11}(A-e,i,m,w)$

$= (1500 + 300 - 20)(1.08) - (10000 + 90)(.015) - (1500 - 60)(.12)$
$- 1750(1 - .015 - .12)$

$\Longrightarrow Pr_{11}(A-e,i,m,w) = 84.5$

$\Longrightarrow G^{(\tau)} = 84.5 - 76.5 = 8$

(b) $$G^{i} = Pr_{11}\begin{pmatrix} A-i \\ E-e,m,w \end{pmatrix} - \underbrace{Pr_{11}\begin{pmatrix} E-i \\ E-e,m,w \end{pmatrix}}_{=76.5 \text{ from } 6(c)}$$

$Pr_{11}\begin{pmatrix} A-i \\ E-e,m,w \end{pmatrix} = (1500 + 300 - 15)(1.08) - (10000 + 100)(.02)$
$- (1500 - 50)(.1) - 1750(1 - .02 - .1)$

$= 40.8$

$\therefore G^{i} = 40.8 - 76.5 = -35.7$

<u>Note</u>: The only thing that changed is the interest rate

$\therefore G^{i} = ({}_{10}V + \pi - e^{\text{expected}})(i^{\text{actual}} - i^{\text{expected}}) = (1500 + 300 - 15)(-.02)$
$= -35.7$ ✓

7) (c) $G^{\{m,e\}} = Pr_{11}\begin{pmatrix} A-m,e \\ E-i,w \end{pmatrix} - Pr_{11}\begin{pmatrix} E-m,e \\ E-i,w \end{pmatrix}$

$$Pr_{11}\begin{pmatrix} A-m,e \\ E-i,w \end{pmatrix} = (1500+300-20)(1.1) - (10000+90)(.015)$$

$$- (1500-60)(.1) - 1750(1-.015-.1)$$

$$= 113.9$$

$Pr_{11}\begin{pmatrix} E-m,e \\ E-i,w \end{pmatrix} = 76.5$ from 6(c)

$$\therefore\ G^{\{m,e\}} = 113.9 - 76.5 = 37.4$$

(d) $G^{\{i,w\}\text{ after }\{m,e\}} = Pr_{11}\begin{pmatrix} A-i,w \\ A-m,e \end{pmatrix} - Pr_{11}\begin{pmatrix} E-i,w \\ A-m,e \end{pmatrix}$

$Pr_{11}\begin{pmatrix} A-i,w \\ A-m,e \end{pmatrix} = 84.5$ (See 7(a))

$$Pr_{11}\begin{pmatrix} E-i,w \\ A-m,e \end{pmatrix} = (1500+300-20)(1.1) - (10000+90)(.015)$$

$$-(1500-60)(.1) - 1750(1-.015-.1)$$

$= 113.9$ (Same as above in part 7(c))

$$\therefore\ G^{\{i,w\}\text{ after }\{m,e\}} = 84.5 - 113.9 = -29.4$$

<u>Note</u>: $G^{(t)} = G^{\{m,e\}} + G^{\{i,w\}\text{ after }\{m,e\}}$

$G^{(t)} = 8$ from 7(a)

$G^{\{m,e\}} = 37.4$ from 7(c)

$G^{\{i,w\}\text{ after }\{m,e\}} = -29.4$ from 7(d)