Solutions to MLCM5S4 Exercises

1) (a) $NPV = -300 + 280v + 48v^2 = 0$

$$\Rightarrow v = \frac{-280 \pm \sqrt{280^2 - 4(48)(-300)}}{2(48)} \Rightarrow v \doteq .92481$$

$$\Rightarrow i = \frac{1}{v} - 1 \doteq 8.13\% = IRR$$

TI BA II Plus Professional Keystrokes:

This line inputs data → [CF] 300 [+/-] [ENTER] [↓] 280 [ENTER] [↓] [↓] 48 [ENTER]

[2nd] [Quit] [IRR] [CPT] ([2nd] [Quit] to get back to calculator mode)

(b) $NPV = -300 + 280v + 48v^2 \qquad v = \frac{1}{1.06}$

$$\Rightarrow NPV = 6.87$$

TI BA II Plus Professional Keystrokes:

Input data as above, then

[2nd] [Quit] [NPV] 6 [ENTER] [↓] [CPT]

(Then [2nd] [Quit] to get back to calculator mode)

DPP: using hurdle rate = .06

$NPV(0) = -300 < 0$

$NPV(1) = -300 + 280v_{.06} < 0$

$NPV(2) = -300 + 280v_{.06} + 48v_{.06}^2 > 0$ ✓

$\therefore DPP = 2$

TI BA II Plus Professional Keystrokes: (calculator refers to DPP as DPB = discounted pay back?)

Input data as above, then

[2nd] [Quit] [NPV] 6 [ENTER] [↓] [↓] [↓] [↓] [CPT] (use next integer larger than result)

(Then [2nd] [Quit] to get back to calculator mode)

2) (a) $\pi_2 = Pr_2 \cdot P_x = 60(.8) = 48$

(b) This is just $Pr_2 = 60$

(c) This is just $\pi_2 = 48$ (see part (a))

3) (a) Since $Pr_0 = -2500$, the pre contract expenses are 2500.

(b) This is just $\pi_5 = Pr_5 \cdot {}_4P_x$

Note ${}_4P_x = e^{-4\mu} = (.9)^4$

$\therefore \pi_5 = 580(.9)^4 = 380.538$

(c) $NPV(5) = \pi_0 + \pi_1 v + \pi_2 \cdot v^2 + \pi_3 \cdot v^3 + \pi_4 \cdot v^4 + \pi_5 \cdot v^5$

Since $d = .1$, then $v = .9$, and note ${}_nP_x = (.9)^n$

Also, $\pi_0 = Pr_0 = -2500$

$\pi_1 = Pr_1 = 1900$

$\pi_2 = Pr_2 \cdot P_x = 250 \cdot (.9)$

$\pi_3 = Pr_3 \cdot {}_2P_x = 360(.9)^2$

$\pi_4 = Pr_4 \cdot {}_3P_x = 470(.9)^3$

$\pi_5 = Pr_5 \cdot {}_4P_x = 580(.9)^4$

$\therefore NPV(5) = -2500 + 1900(.9) + 250(.9)^3 + 360(.9)^5 + 470(.9)^7 + 580(.9)^9$

$\doteq 54.3298$

Note: You can use the TI BA II Plus Professional as in #1, but you must convert $d = .1$ to $i = 11.11\%$. Input the data $\pi_0, \pi_1, \ldots$, up to and including π_5 and proceed as in #1.

4) (a) The IRR is the interest rate that makes $NPV = 0$.

$$NPV = -700 + 120v + 125v^2 + 130v^3 + 130v^4 + 130v^5 + 130v^6 + 130v^7 + 125v^8 + 120v^9 + 110v^{10}$$

$NPV = 0$ gives an unsolvable equation, but we can use the TI BA II Plus Professional to solve numerically as follows:

[CF] [2nd] [CLR WORK] (clears previous CF entries)

We can input the Π_k-values one at a time, but since some values are repeated, we can proceed as follows:

[CF] 700 [+/-] [ENTER] [↓] 120 [ENTER] [↓] [↓] 125 [ENTER] [↓] [↓]
130 [ENTER] [↓] 5 [ENTER] [↓] 125 [ENTER] [↓] [↓] 120 [ENTER] [↓] [↓]
110 [ENTER] [2nd] [Quit]

Then [IRR] [CPT].

$\therefore$ IRR $\doteq 12.33399\%$

(b) With data entered as above,

[NPV] 8 [ENTER] [↓] [CPT]

$\therefore$ NPV $\doteq 141.7969$

(c) $PM = \dfrac{NPV}{EPV(\text{Premiums})}$ using RDR (HR)

$$\Rightarrow PM \doteq \frac{141.7969}{1250(8)} \doteq .01418$$

(d) DPP = smallest integer k such that $NPV(k) \geq 0$

Use guess & check or the TI BA II Plus Professional as follows: with data entered as in part (a), then [NPV] 8 [ENTER] [↓] [↓] [↓] [↓] [CPT]
The result is 7.54369..., and so the DPP = 8. You should check, by hand or with the calculator, that $NPV(7) < 0$ and $NPV(8) > 0$.

5) The profit signature is $\pi = (-1000, 750, 200 \cdot {}_1p_x, 200 \cdot {}_2p_x)$

$${}_np_x \overset{\text{by iii}}{=\!=} p^n = e^{-\mu \cdot n}$$

$$\therefore \pi = (-1000, 750, 200e^{-\mu}, 200e^{-2\mu})$$

$$\therefore NPV = 17.49 = -1000 + 750v + 200e^{-\mu} \cdot v^2 + 200e^{-2\mu} \cdot v^3$$

$v = \frac{1}{1.05} \Rightarrow$ The above equation is quadratic in $e^{-\mu}$

$$\left.\begin{aligned} a &= 200v^3 \\ b &= 200v^2 \\ c &= -1000 + 750v - 17.49 \end{aligned}\right\} \Rightarrow e^{-\mu} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow e^{-\mu} \doteq .9 = p \Rightarrow q_x = q = 1 - p = .1$$

6) As above, the profit signature is $\pi = (-1000, 750, 200p, 200p^2)$

$IRR = .06 \Rightarrow NPV = 0$ using $i = .06$

$$\therefore NPV = -1000 + 750v_{.06} + 200p \cdot v_{.06}^2 + 200p^2 \cdot v_{.06}^3 = 0 \quad \left(\begin{array}{l}\text{quadratic in} \\ p = e^{-\mu}\end{array}\right)$$

$$\left.\begin{aligned} a &= 200v_{.06}^3 \\ b &= 200v_{.06}^2 \\ c &= -1000 + 750v_{.06} \end{aligned}\right\} \Rightarrow p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow p \doteq .89214 \Rightarrow q = 1 - p = .10786$$

7) (a) $X = \Pi_1 = Pr_1 = 70$

(b) $Y = Pr_2$; $\Pi_2 = 77 = Pr_2 \cdot P_x = Y(.9625) \Rightarrow Y = \frac{77}{.9625} = 80$

(c) $\Pi_4 = Pr_4 \cdot {}_3P_x \Rightarrow {}_3P_x = \frac{99}{116}$

(d) As in part (c), ${}_2P_x = \frac{\Pi_3}{Pr_3} = \frac{88}{96}$ (Note: not needed here) ${}_4P_x = \frac{\Pi_5}{Pr_5} = \frac{55}{72}$

${}_3P_x = {}_2P_x \cdot P_{x+2} \Rightarrow P_{x+2} = \frac{99/116}{88/96} = \frac{\cancel{96}^{3}(99)^{9}}{\cancel{88}_{1}(\cancel{116}\ 29)} = \frac{27}{29}$

$\therefore q_{x+2} = \frac{2}{29}$

(e) $NPV(3) = -230 + \overbrace{70}^{=X} \cdot v + 77v^2 + 88v^3$

$v = \frac{1}{1.1} \Rightarrow NPV(3) = -36.61157$

Note: As in previous problems, we could've used the TI BA II Plus Professional.

(f) $NPV(3) = -36.61157 < 0$

$NPV(4) = NPV(3) + 99v^4 \doteq 31.00676 > 0$

$\therefore DPP = 4$

Note: We could've used the TI BA II Plus Professional here also.

(g) $PM = \frac{NPV}{EPV(Premiums)}$ using $i = 10\%$

$NPV = NPV(5) = NPV(4) + 55v^5 = 65.\cancel{55}^{157}$ (or use)

$EPV(Premiums) = 285 \cdot \ddot{a}_{x:\overline{5}|} = 285(1 + vP_x + v^2 \cdot {}_2P_x + v^3 \cdot {}_3P_x + v^4 \cdot {}_4P_x)$

$v = \frac{1}{1.1}$ See above for P_x, ${}_2P_x$, ${}_3P_x$, and ${}_4P_x$

$\therefore EPV(Premiums) \doteq 285(3.79553) = 1081.726$

$\Rightarrow PM \doteq \frac{65.157}{1081.726} \doteq .0602$