Solutions to MLCM556 Exercises

Abbreviations:

NRA = Normal Retirement Age

OFP = Optional Form of Payment

ERA = Early Retirement Age

NRB = Normal Retirement Benefit

NFP = Normal Form of Payment

ERB = Early Retirement Benefit

ERF = Early Retirement Factor

1)

Age	Salary
25	40000
26	40000(1.03)
⋮	⋮
59	$40000(1.03)^{34}$
⋮	⋮
64	$40000(1.03)^{39}$

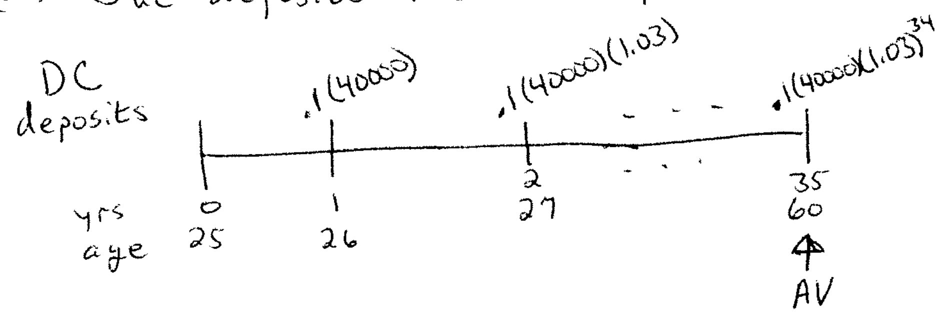
Employee (ee) Contribution $\le 6\%$

$\Longrightarrow$ company match $= \frac{1}{2}$(ee contribution)

ee contribution $> 6\%$

$\Longrightarrow$ company match $= 3\%$

(a) Sue deposits 7% $\Longrightarrow$ company match = 3% $\Longrightarrow$ total deposit = 10%



Note: Sue's final year salary $= 40000(1.03)^{34}$

$$AV \overset{VEP}{=} .1(40000)(1.03)^{34} + .1(40000)(1.03)^{33}(1.06) + \cdots \text{ (35 terms)}$$

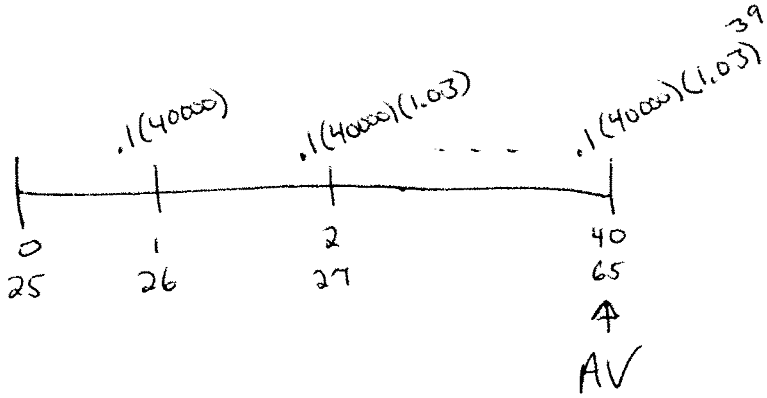
$$= .1(40000)(1.03)^{34}\left[1 + \frac{1.06}{1.03} + \cdots\right] = .1(40000)(1.03)^{34} \cdot s_{\overline{35}|\left(\frac{1.06}{1.03}-1\right)}$$

Converting to a benefit we get $AV = B_{60} \cdot \ddot{a}_{60}$

$$\Longrightarrow B_{60} \overset{ILT}{=} \frac{AV}{11.1454} = \cancel{58,286.82}\ 58,286.82$$

$$\therefore RR = \frac{B_{60}}{\text{final yr salary}} \doteq .533 \quad (53.3\%)$$

1) (b) total deposit = 10% of salary each year as in (a)



Note: Sue's final year salary $= 40000(1.03)^{39}$

$$AV \overset{VEP}{=} .1(40000)(1.03)^{39} + .1(40000)(1.03)^{38}(1.06) + \dots \text{ (40 terms)}$$

$$= .1(40000)(1.03)^{39}\left[1 + \frac{1.06}{1.03} + \dots\right] = .1(40000)(1.03)^{39} s_{\overline{40}|\left(\frac{1.06}{1.03}-1\right)}$$

Converting to a benefit, we get $AV = B_{65} \cdot \ddot{a}_{65}$

$$\Longrightarrow B_{65} \overset{ILT}{=} \frac{AV}{9.8969} = 94{,}624.65$$

$$\therefore RR = \frac{B_{65}}{\text{final yr salary}} = .747 \quad (74.7\%)$$

(c) Sue deposits 5% $\Longrightarrow$ company match = 2.5%

$\Longrightarrow$ total deposit = 7.5% of salary each year

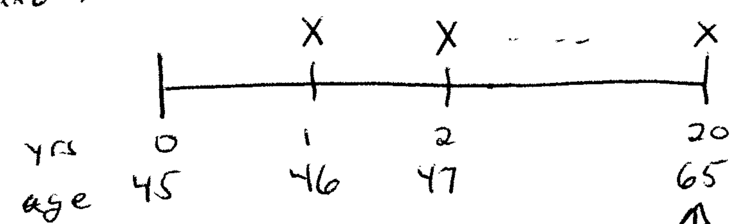
We could go through the calculations in part (b) again, but it is quicker to recognize that we'll get the answer here by multiplying the answer in part (b) by $\frac{.075}{.1}$.

$$\therefore RR = \frac{.075}{.1}(.747) = .560 \quad (56.0\%)$$

2)

Age	Salary
45	50000
46	50000(1.04)
⋮	⋮
64	$50000(1.04)^{19}$ = Tom's final year salary

DC Account:



$AV = X\, s_{\overline{20}|.09}$

$$RR = .35 = \frac{\text{Annual Ret. Benefit}}{50000(1.04)^{19}}$$

$$\Rightarrow \text{Annual Ret. Benefit} = B = .35(50000)(1.04)^{19}$$

Also $AV = X \cdot s_{\overline{20}|.09} = B \cdot \ddot{a}_{65}$

$$\therefore\ X \cdot s_{\overline{20}|.09} = \underbrace{.35(50000)(1.04)^{19}}_{=B} \cdot \underbrace{10}_{=\ddot{a}_{65}}$$

$$\Rightarrow X = 7206.76$$

Since the company matches dollar for dollar up to 2500, Tom needs to deposit $X - 2500 = 4706.76$ each year.

3) As in #2, $X = 7206.76$. For Tom to get the maximum company match, he would have to deposit 5000, but then the total deposit would be $5000 + 2500 = 7500 > X$.

Since for every 3 total dollars deposited, Tom contributes 2 and the company contributes 1, the company match ~~should~~ accounts for 1/3 of the total deposit and Tom's contribution accounts for 2/3 of the total deposit.

$\therefore$ Tom needs to deposit $\frac{2X}{3} \doteq \cancel{2402}$ 4804.51 each year.

4) Let X = Omar's monthly benefit

$$\therefore \quad 50000\,\ddot{a}_{65} = 12X \cdot \ddot{a}^{(12)}_{65} \qquad \ddot{a}_{65} \overset{ILT}{=} 9.8969$$

(a) $\ddot{a}^{(12)}_{65} \overset{UDD}{=} \alpha(12)\,\ddot{a}_{65} - \beta(12) \overset{ILT}{=} (1.00028)(9.8969) - .46812$

$$\Rightarrow X = 4372.25$$

(b) $\ddot{a}^{(12)}_{65} \overset{\text{2-term}}{\underset{WH}{=}} \ddot{a}_{65} - \frac{11}{24} \Rightarrow X = 4369.00$

(c) $\ddot{a}^{(12)}_{65} \overset{\text{3-term}}{\underset{WH}{=}} \ddot{a}_{65} - \frac{11}{24} - \frac{143}{1728}(\mu_{65} + \delta) \qquad {}_2p_{64} = e^{-2\mu_{65}} = \frac{l_{66}}{l_{64}}$

$$\delta = \ln(1+i) = \ln(1.06)$$

$$\Rightarrow X = 4372.02$$

5) Kim has 35 complete years of service.

(a) $NRB = 1800(35) = 63000$ (payable annually in advance for life)

(b) $LS = 63000 \cdot \ddot{a}_{65} \overset{ILT}{=} 63000(9.8969) = 623{,}504.70$

(c) For 10-year C&L, $63000 \cdot \ddot{a}_{65} = X \cdot \ddot{a}_{\overline{65:\overline{10|}}} = X(\ddot{a}_{\overline{10|}} + {}_{10|}\ddot{a}_{65})$

$$\therefore 63000\,\ddot{a}_{65} = X(\ddot{a}_{\overline{10|}} + {}_{10}E_{65} \cdot \ddot{a}_{75})$$

$\ddot{a}_{65} \overset{ILT}{=} 9.8969 \qquad \ddot{a}_{75} \overset{ILT}{=} 7.217$

${}_{10}E_{65} \overset{ILT}{=} .39994 \qquad \ddot{a}_{\overline{10|}.06} \overset{TVM}{=} 7.8017$

$\Longrightarrow X = 58336.53$

For Joint and 100% Survivor, $63000\,\ddot{a}_{65} = X \cdot \ddot{a}_{\overline{65:65}}$

$$\ddot{a}_{\overline{65:65}} = \ddot{a}_{65} + \ddot{a}_{65} - \ddot{a}_{65:65} \overset{ILT}{=} 2(9.8969) - 7.8552$$

$\Longrightarrow X = 52225.95$

For Joint & 50% Survivor, $63000\,\ddot{a}_{65} = \overbrace{X \cdot \ddot{a}_{65:65}}^{\text{both alive}} + \overbrace{X(\ddot{a}_{65} - \ddot{a}_{65:65})}^{\text{Kim alive, David dead}}$

$$+ \underbrace{\tfrac{1}{2}X(\ddot{a}_{65} - \ddot{a}_{65:65})}_{\text{David alive, Kim dead}}$$

$\ddot{a}_{65:65} = 7.8552$

$\ddot{a}_{65} = 9.8969$

$\Longrightarrow X = 57109.27$

For Joint & 50% Contingent, $63000\,\ddot{a}_{65} = X \cdot \ddot{a}_{65:65} + \tfrac{1}{2}X(\ddot{a}_{65} - \ddot{a}_{65:65})$

$$+ \tfrac{1}{2}X(\ddot{a}_{65} - \ddot{a}_{65:65})$$

$\Longrightarrow X = 63000$

↳ since both are same age, 65.

<u>Remark</u>: The Joint & 50% Survivor / Contingent terminology is <u>not</u> universal and not on the exam. They will explain the terminology as I did in this problem.

6) They both terminated employment with 3yrs of service. According to Jim's plan, he is 0% vested. So for Jim, $APV = 0$.

According to Tim's plan, he is 60% vested.

$\therefore$ his vested benefit is $(.6)(5000) = 3000$, and

$$APV_{43}^{Tim} = 3000 \cdot {}_{22|}\ddot{a}_{43} = 3000 \cdot {}_{22}E_{43} \cdot \ddot{a}_{65}$$

$$\ddot{a}_{65} \overset{ILT}{=} 9.8969 \quad \text{and} \quad {}_{22}E_{43} = v^{22} \cdot {}_{22}P_{43} = v_{.06}^{22} \cdot \frac{l_{65}}{l_{43}}$$

$$\Longrightarrow APV_{43}^{Tim} = 6725.38$$

7) William's annual benefit amount is

$$B = .02S_1 + .02S_2 + \cdots + .02S_{30}, \text{ where}$$

S_k = salary earned during the k^{th} year working

$$\therefore B = .02(S_1 + S_2 + \cdots + S_{30})$$

$$= .02 \cdot (\text{total salary earned over his working lifetime})$$

$$= .02(1{,}500{,}000) = 30{,}000$$

William is paid 30000 @ BOY for life, starting at NRA.

8) (a) Cindy's accrued benefit at age 45

$= B_{45} = .015\,(600000) = 9000$

$\therefore$ if she terminates employment now, she would receive $\frac{9000}{12} = 750$ at the beginning of each month for her lifetime, starting at age 65.

(b) $APV_{45}^{AB} = 12(750) \cdot {}_{20|}\ddot{a}_{45}^{(12)} = 9000 \cdot {}_{20}E_{45} \cdot \ddot{a}_{65}^{(12)}$ → Accrued Benefit

$= 9000\, v^{20} \cdot {}_{20}P_{45} \cdot \ddot{a}_{65}^{(12)} = 9000(1.04)^{-20}\,(.9)\,(11.4)$

$\therefore APV_{45}^{AB} = 42{,}142.77$

(c) If Cindy works to age 65, then her total salary is

$S = 600000 + S_{45} + \underbrace{S_{46}}_{=S_{45}(1.03)} + \cdots + \underbrace{S_{64}}_{=S_{45}(1.03)^{19}}$

$\therefore S = 600000 + \underbrace{S_{45}}_{=70000} \underbrace{(1 + 1.03 + \cdots + (1.03)^{19})}_{= s_{\overline{20|}.03}}$

$\therefore S = 2480926.21$

$\Longrightarrow B_{65} = .02\,(S) = 49618.52$ → annual #; she actually gets $\frac{49618.52}{12}$ per month

$\therefore APV_{45}^{RB} = 49618.52 \cdot {}_{20|}\ddot{a}_{45}^{(12)} \overset{\text{see above}}{=} 232{,}340.21$ → Retirement Benefit

Note $APV_{45}^{RB} = \frac{APV_{45}^{AB}}{9000}\,(49618.52)$

(d) Cindy's final year salary is $70000(1.03)^{19}$!

$\therefore RR = \frac{49618.52}{70000\,(1.03)^{19}} \doteq .404$ (40.4%)

9)

Age	Salary
35	40000
36	40000(1.05)
⋮	⋮
64	$40000(1.05)^{29}$

$$B_{65} = .015 \cdot S_{64} \cdot \underbrace{(30)}_{\text{yrs of service at retirement}}$$

$$= .015\left[40000(1.05)^{29}\right](30)$$

$$\therefore\ B_{65} = 74090.44$$

10) Donna's final year salary is $Y(1+.01x)^{29}$

$$B_{65} = .015\left[Y(1+.01x)^{29}\right]\underbrace{(30)}_{\text{yrs of service at retirement}}$$

$$\therefore\ RR = \frac{.015\left[\cancel{Y(1+.01x)^{29}}\right](30)}{\cancel{Y(1+.01x)^{29}}} = .015(30) = .45$$

Remark: Since this is a final year salary plan, we didn't need Y or x to get an answer. Note this would be the RR for Jamie in problem #9.

11)

x = Age	Salary $= S_x$
45	$80000 = S_{45}$
46	$80000(1.03) = S_{46}$
$\vdots$	$\vdots$
57	$80000(1.03)^{12}$
58	$80000(1.03)^{13}$
59	$80000(1.03)^{14}$
$\vdots$	$\vdots$
62	$80000(1.03)^{17}$
63	$80000(1.03)^{18}$
64	$80000(1.03)^{19}$

Remark: There are several ways to sum S_K, S_{K+1}, & S_{K+2}. E.g. $S_{57} + S_{58} + S_{59} = ?$

Method 1: (Just do it & it's only 3yrs)

Method 2: $S_{57} + S_{58} + S_{59}$

$= S_{57}\left(1 + 1.03 + (1.03)^2\right)$

$= S_{57} \cdot s_{\overline{3}|.03}$

Method 3: $S_{57} + S_{58} + S_{59} = S_{59} + S_{58} + S_{57}$

$= S_{59}\left(1 + \frac{1}{1.03} + \frac{1}{(1.03)^2}\right) = S_{59}(1 + v + v^2)$

$= S_{59} \cdot \ddot{a}_{\overline{3}|.03}$

(a) termination at age 60 $\Rightarrow$ 15 years of service

final 3-year average salary $= \frac{1}{3}(S_{57} + S_{58} + S_{59})$

$= 117516.92$

$\therefore B_{60} = .02(100000)(15) + .03(17516.92)(15) = 37882.61$

Note: This is an annual #. Lou actually would receive $\frac{37882.61}{12}$ at the beginning of each month, starting at age 65.

(b) retirement at age 65 $\Rightarrow$ 20 yrs of service

final 3-year average salary $= \frac{1}{3}(S_{62} + S_{63} + S_{64}) = 136{,}234.31$

$\therefore B_{65} = .02(100000)(20) + .03(36234.31)(20) = 61740.59$

(c) $APV_{65} = 61740.59 \cdot \ddot{a}_{65}^{(12)} = 753{,}235.20$

12) Since Lou retires at age 60, his age 65 benefit is $B_{60} \underset{\#11(a)}{\overset{\text{see}}{=}} 37882.61$. We must reduce this amount by the ERF if he starts his benefit at age 60. Since ERB's are determined by actuarial equivalence, we have

$$B_{60} \cdot {}_{5|}\ddot{a}_{60}^{(12)} = X \cdot \ddot{a}_{60}^{(12)}$$ where $X =$ annual retirement benefit starting at age 60 (still payable monthly)

$\Rightarrow$ both sides are APV_{60} of retirement benefit

$$LHS = APV_{60}^{AB}$$

$$RHS = APV_{60}^{ERB}$$

$$\therefore (37882.61)\, {}_5E_{60} \cdot \ddot{a}_{65}^{(12)} = X\, \ddot{a}_{60}^{(12)}$$

$${}_5E_{60} = v_{.04}^5 \cdot {}_5p_{60} = (1.04)^{-5}(.95)$$

$$\ddot{a}_{65}^{(12)} \overset{\#11}{=} 12.2$$

$$\ddot{a}_{60}^{(12)} = 13.6$$

$$\Longrightarrow X = 26534.92 \text{ (annual ERB)}$$

$$\Longrightarrow \text{Lou's monthly ERB} = \frac{26534.92}{12} = 2211.24$$

13)

Age	Salary
35	60000
36	60000(1.04)
⋮	⋮
60	$60000(1.04)^{25}$
61	$60000(1.04)^{26}$
62	$60000(1.04)^{27}$
63	$60000(1.04)^{28}$
64	$60000(1.04)^{29}$

Avg = 173268.35
= Don's final 5-year average salary

<u>Note</u>: Ron's final 5-year average salary
$= (1.04)^{-5} \cdot$ Don's final 5-year average salary
$= 142413.95$

(a) $B_{60}^{Ron} = .02(142413.95)\overset{\text{yrs of service}}{(25)} = 71206.98$ (annual #, payable at age 65)

pension reduction factor $= .06(5) = .3 \Rightarrow ERF = .7$

$\therefore\ ERB_{60}^{Ron} = .7(71206.98) = 49844.89$ annually,

or $\dfrac{49844.89}{12} = 4153.74$ monthly

(b) $B_{65}^{Don} = .02(173268.35)\overset{\text{yrs of service}}{(30)} = 103961.01$ annually,

or $\dfrac{103961.01}{12} = 8663.42$ monthly

(c) $APV_{60}^{\text{Ron's RB}} = 49844.89 \cdot \ddot{a}_{60}^{(12)} = 677890.50$

(d) $APV_{65}^{\text{Don's RB}} = 103961.01 \cdot \ddot{a}_{65}^{(12)} = 1,268,324.32$