3/14/19

MS 55: Incorporating Expenses

Types of Expenses

1) Settlement Expense (paid at time the benefit is paid)

2) "Premium Type" (usually payable at BOY when premiums are paid)
   A) per policy (flat $ amount)
      (paid for lifetime of policy)
   B) per dollar insured (per $1000 insured)
      (paid for lifetime of policy)
   C) percent of premium
      (paid for premium payment period)

Example: (See next page)
For a fully discrete whole life insurance of 500,000 issued to (30), you are given:

(i) \( A_{30} = 0.1 \)
(ii) \( d = 0.06 \)
(iii) per policy expenses are 500 in the first year and 100 in renewal years
(iv) per $1000 expenses are 1 in the first year and 0.25 in renewal years
(v) percent of premium expenses are 75\% if the first year and 5\% thereafter
(vi) there is a settlement expense of 5000

Determine the annual gross premium using the equivalence principle.

Solution: \[ \text{APV (Ben + Exp)} = \frac{EP}{\text{APV (} \Pi^g \text{)}} \]

\[ \text{APV (Ben + Exp)} = 505000 \times A_{30} \]
\[ + (225 + 0.05 \times 250) \times \ddot{a}_{30} + 775 + 7 \times \Pi^g \]

\[ 505000 A_{30} + 225 \ddot{a}_{30} + 0.05 \Pi^g \ddot{a}_{30} + 775 + 7 \Pi^g = \Pi^g \ddot{a}_{30} \]

\[ A_{30} = 0.1 \implies \ddot{a}_{30} = \frac{1 - 0.1}{0.06} = 15 \]
\[ d = 0.06 \]
\[ \therefore \Pi^g = 4033.21 \]
Note: \( APV(\text{Ben} \& \text{Exp}) = APV(\text{Ben}) + APV(\text{Exp}) \)

\( APV(\text{Ben}) = APV(\text{net premiums} = \Pi^n) \)

Define \( APV(\text{Exp}) = \) the expense premium (load) by \( \Pi^e = \Pi^g - \Pi^n \)

\( \therefore APV(\text{Exp}) = APV(\Pi^e) \)

When asked to calculate \( \Pi^e \), we have two methods:

1) \( \Pi^e = \Pi^g - \Pi^n \) (easier in cases in which there is a percent of premium exp.

2) \( APV(\text{Exp}) = APV(\Pi^e) \)

Example: See the previous page. Determine the expense load.

\( \Pi^g = 4033.21 \)

\( \Pi^n = \frac{500000A_{30}}{\Delta_{30}} = \frac{500000(1.1)}{15} = 3333.33 \)

\( \therefore \Pi^e = 699.88 \rightarrow \) amount paid at ROY to pay expenses on average.

Remark: \( \Pi^e_0 = 4024.91 \) (time 0 actual expense) actual renewal expenses = \( \Pi^e_1 = 426.62 \)
**M355 Exercises**

#3)

\[ APV(Ben_{4}\text{Exp}) = 100000 \ddot{A}_{35} + (275 + 1 \pi t) \cdot \ddot{a}_{35} \]

\[ APV(\text{prem}) = \pi \cdot \ddot{a}_{35} \]

\[ \therefore 100000 \ddot{A}_{35} + 275 \ddot{a}_{35} = 9 \pi \ddot{a}_{35} \]

\[ \therefore \pi = \frac{100000}{9} \cdot \frac{\ddot{A}_{35}}{\ddot{a}_{35}} + \frac{275}{9} = 1234.44 \text{ (gross)} \]

\[ \text{net premium} = \frac{100000 \ddot{A}_{35}}{\ddot{a}_{35}} = 836 \]

\[ \therefore \Pi^e = 1234.44 - 836 = 398.44 \]
\[\text{APV (B \& Exp)} = 100000 A_x + 102 \pi + 10200 a_{x:141} + 5 + 5 \cdot \ddot{a}_x\]

\text{Given:} \quad A_x = 0.5148197 \quad \& \quad a_{x:141} = 10.35 \quad \& \quad d = 0.02913

\[\therefore \dot{a}_x = 1 - A_x\]

\[\text{APV (gross prem)} = \pi \cdot \ddot{a}_{x:151} = \pi \cdot (1 + a_{x:141})\]

\text{Solve for } \pi