3/26/19:

Test 10 tomorrow covers sections 41, 5, & 6 from Module 3.

M3S6 Example

(See next page)
Module 3: Premiums
Section 6: Refunding Premiums

Example:

For a 20-year deferred annuity due issued to (45) paying $100,000 per year, you are given

i) Premiums are paid at the beginning of each year while alive, for at most 10 years
(ii) The first year percent of premium expense is 60%; thereafter it's 5%
(iii) Premiums are refunded, without interest, at the end of the year of death if (45) dies within the 20-year deferred period

Set up an equation to solve for the gross annual premium, using the equivalence principle.

\[
\text{Solution: } \text{APV}(\text{Ben } + \text{Exp}) = \text{APV}(\text{premium})
\]

\[
= \pi \cdot \dot{a}_{45:101}
\]

\[
\text{APV} = 100000 \cdot e_{45} \cdot \dot{a}_{65} + .55 \pi + .05 \pi \cdot \dot{a}_{45:101}
\]

\[+ \pi \cdot (IA)_{45:101} + 10\pi \cdot e_{45} \cdot A_{55:101}\]

\[
\pi \cdot \dot{a}_{45:101} = 100000 \cdot e_{45} \cdot \dot{a}_{65} + .55 \pi + .05 \pi \cdot \dot{a}_{45:101} + \pi (IA)_{45:101} + 10\pi \cdot e_{45} \cdot \dot{A}_{55:101}\]
M357: Percentile (Probability) Premiums

Example: \[ \frac{41}{\text{M357 Exercises}} \]
Note: Insurer receives 460 in premiums.

Q: Pr(\( S < 460 \)) where \( S = \sum_{i=1}^{100} (10Z_x)_i \)

\[ E[S] = 100 \cdot 10 \cdot \bar{A}_x = 1000 \cdot \frac{\mu}{m+8} = 400 \]

\[ \text{Var}(S) = 100 \cdot 10^2 \cdot (2\bar{A}_x - (\bar{A}_x)^2) = 10000 \left( \frac{\mu}{m+26} - \left( \frac{\mu}{m+8} \right)^2 \right) \]

\[ \Rightarrow \text{Var}(S) = 900 \]

\[ \therefore \text{Pr}(S < 460) = \text{Pr}(\text{SND} < \frac{460-400}{\sqrt{900}}) \]

\[ = \text{Pr}(\text{SND} < 2) = 0.9772 \]

More examples: (See next page.)
Module 3: Premiums
Section 7: Percentile (Probability) Premiums

Example 1:

For a block of 30 fully discrete whole life insurances of 1 on independent lives age $x$, you are given:

i) $i = 0.06$
ii) $A_x = 0.249$
iii) $\hat{A}_x = 0.095$
iv) $\pi = 0.025$

Using the normal approximation, determine the probability of a positive total loss on the policies issued.

**Solution:**

$T = \sum_{i=1}^{30} (oL)_i$

$Q: \Pr(T > 0) = \Pr \left( \text{SND} > \frac{0 - E[T]}{\sqrt{Var(T)}} \right)$

$E[T] = 30 \cdot E[oL]$ \hspace{1cm} $Var(T) = 30 \cdot Var(oL)$

$oL = Z_x - 0.025 \cdot \bar{Y}_x$

For $E[oL]$, we have $E[oL] = A_x - 0.025 \cdot \bar{a}_x = -0.0826...$

$\bar{a}_x = \frac{1 - A_x}{i(1+i)}$

For $Var(oL)$, $oL = (1 + \frac{0.025}{i}) Z_x - #$

$\bar{Y}_x = \frac{1 - 2A_x}{i}$ \hspace{1cm} $\Rightarrow Var(oL) = \left( 1 + \frac{0.025}{i} \right)^2 \left( \hat{A}_x - (A_x)^2 \right)$

$= 0.0685... \hspace{1cm} \text{[2]}$

$\therefore E[T] = -2.48075 \hspace{1cm} Var(T) = 2.057... \hspace{1cm} \text{[4]}$

$\therefore \Pr(T > 0) = \Pr \left( \text{SND} > 1.729... \right) \frac{\text{SND}}{\text{Table}} \approx 0.0418$