M357: (Continued) Percentile (Probability) Premiums

There are two types of questions

1) for a lot of iid policies. In this case, use normal approximation techniques (See previous notes)

2) for an individual policy (See examples on the next pages.)
Module 3: Premiums
Section 7: Percentile (Probability) Premiums

Example 2:

Actuarial assumptions are \( CF(\mu = 0.03, \delta = 0.06) \). For a fully continuous whole life insurance of 500 issued to \( x \), determine annual premium rate such that the probability of a positive loss-at-issue is 0.05.

\[
\text{Solution: } \quad oL = 500 \overline{z}_x - \pi \cdot \overline{y}_x = 500 \overline{u}^T - \pi \cdot \left( \frac{1 - \overline{u}^T}{\delta} \right)
\]

\[
= 500 \overline{u}^T - \pi \cdot \left( \frac{1 - \overline{u}^T}{\delta} \right)
\]

\[
\therefore \quad oL = (500 + \frac{\pi}{\delta}) \cdot \overline{u}^T - \frac{\pi}{\delta}
\]

\[
\therefore \quad \Pr(oL > 0) = 0.05
\]

\[
\Rightarrow \quad \Pr(\overline{u}^T > \frac{\pi}{500 \delta + \pi}) = 0.05, \quad \overline{u}^T = e^{-\delta t}
\]

\[
\Rightarrow \quad \Pr(\overline{T} < \frac{\ln \left( \frac{\pi}{500 \delta + \pi} \right)}{-\delta}) = 0.05
\]

\[
\therefore \quad t^b_x = 0.05 \Rightarrow t^p_x = e^{-\mu t} = 0.95
\]

\[
\Rightarrow \quad t = \frac{\ln(0.95)}{-\mu}
\]

\[
\therefore \quad \frac{\ln \left( \frac{\pi}{500 \delta + \pi} \right)}{-\delta} = \frac{\ln(0.95)}{-\mu} \Rightarrow \ln \left( \frac{\pi}{500 \delta + \pi} \right) = \ln(0.95)^2
\]

\[
\Rightarrow \quad \frac{\pi}{500 \delta + \pi} = (0.95)^2 \Rightarrow \pi = 277.67
\]
Module 3: Premiums
Section 7: Percentile (Probability) Premiums

Example 3:

For a fully discrete whole life insurance of 1000 issued to (x), you are given:

i) \( i = 0.06 \)

ii) \( n \bar{q}_x = 0.02n \) for \( 0 \leq n \leq 50 \)

iii) \( \pi \) is the smallest premium such that \( \text{Pr}(\bar{0}L > 0) \) is at most 0.25.

Determine \( \pi \).

\[
\begin{align*}
\bar{0}L &= 1000 \cdot Z_x - \pi \cdot \bar{Y}_x \\
&= 1000 \cdot 2^{K+1} - \pi \cdot \bar{a}^{K+1}
\end{align*}
\]

\[
\text{Pr}(\bar{0}L > 0) = t \bar{b}_x \leq 0.25
\]

\[
\Rightarrow 0.02t \leq 0.25
\]

\[
\Rightarrow t \leq 12.5
\]

\[
K = 12 \Rightarrow 12 \leq t < 13
\]

\[
K = 12 \Rightarrow \bar{0}L < 0
\]

\[
1000 \cdot 2^{13} - \pi \cdot \bar{a}^{13} < 0
\]

\[
\Rightarrow \pi > \frac{1000 \cdot 2^{13}}{\bar{a}^{13}} = 49.96 \ldots
\]

Remark: If we tried to finish the problem by using

\[
K = 11 \Rightarrow \bar{0}L > 0
\]

\[
1000 \cdot 2^{12} - \pi \cdot \bar{a}^{13} > 0 \Rightarrow \pi < \frac{1000 \cdot 2^{12}}{\bar{a}^{13}} = 55.92 \ldots
\]

55.92... is the largest premium such that \( \text{Pr}(\bar{0}L > 0) > 0.25 \)
Module 4: Reserves

Idea: Policy is issued at age \( x \).

The premium is set at that time.

Suppose \((x)\) is now \((x+k)\).

Then \( APV_{FB} \) at age \( x+k \)

is more than the \( APV_{FP} \) at age \( x+k \).

The difference is the reserve at age \( x+k \).

I.e. (notation)

\[
_x V = APV_{x+k}(FB) - APV_{x+k}(FP)
\]

reserve at

time \( k \) (age \( x+k \))

M4S2: (Prospective) Loss-at-Time \( k \) PVRV.

Notation:

\[
_k L = rvr \ (prospective) \ loss \ PVRV
\]

\[
_k L = PV_{FB \ RV} - PV_{FP \ RV} \ (at \ age \ x+k)
\]
Module 4: Reserves
Section 2: (Prospective) Loss-at-Time t Present Value Random Variables:
and Expectations (Reserves)

Example 1:

A 20-year endowment insurance issued to (42) pays 1000 at the moment of death. The insurance is purchased with a single premium of 400. Using SULT actuarial assumptions and assuming UDD, determine:

(a) an expression for $1_0L$
(b) the reserve at time 10, $1_0V$
(c) the value of $1_2L|T_{42} = 17.2$
(d) the value of $2_0L$

\[
\begin{align*}
\text{(a)} & \quad 1_0L = 1000 \overline{A}_{52:101} - D \\
\text{(b)} & \quad 1_0V = E \overline{L}_{10} = 1000 \overline{A}_{52:101} = \text{APV}_{52} \text{(FB)} - \text{APV}_{52} \text{(FP)} \\
\text{(c)} & \quad \overline{A}_{52:101} = \frac{i}{\delta} \cdot \overline{A}_{52:101} + 1_0E_{52} \\
\text{(d)} & \quad 2_0L = 1000 \quad \text{(the pure endowment paid to} \\
& \quad \text{(62) is a future benefit at age 62.)}
\end{align*}
\]