M159: Multi-State Models

Multi-State Probabilities:

\( i \quad \xrightarrow{i,j} \quad j \quad \xleftarrow{i,j} \quad k \)

\( n_{ij} = \Pr( (x) \text{ will be in state } j \text{ at age } x+n \mid (x) \text{ is in state } i \text{ at age } x ) \)

\( n_{ii} = \Pr( (x) \text{ remains in state } i \text{ for } n \text{ years} ) \)

Example:

\[ n_{ii}^{(x)} = e^{-\int_{0}^{x} \mu_{x+t} dt} \]

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Generally, \( n_{ii} = e^{-\int_{0}^{x} \mu_{x+t} dt} \)

\[ \mu_{x+t} = \sum_{i,j} n_{ij} \text{ book } \mu_{x+t} \]

Note: \( \mu_{x+t}^{ii} \) does not exist
Specific Models

"Permanent" Disability model

\[ \mu^o_x \neq 0 \]

\[ \begin{array}{c}
(0) \\
(1) \\
(2)
\end{array} \]

\[ nP_{20}^x = 0 = nP_{21}^x = nP_{10}^x \quad nP_{22}^x = 1 \]

\[ nP_{11}^x = e^{-\int_0^x \mu^o_{x+t} \, dt} \quad (nP_{11}^x = nP_{11}^x \text{ since if state 1 is left, then can't return to it}) \]

\[ nP_{00}^x = e^{-\int_0^x (\mu^o_{x+t} + \mu^o_{x+t}) \, dt} \quad (nP_{00}^x = nP_{00}^x) \]

\[ nP_{12}^x = ? \text{ easy way} \]

\[ nP_{12}^x + nP_{11}^x + nP_{10}^x = 1 \]

\[ nP_{12}^x = 1 - nP_{11}^x \]