Recall the "permanent disability model" PDM

\[
\begin{align*}
\text{Probabilities:} & \\
n P_x^{00} &= n P_x^{00} = e^{-S_0 (\mu_1^{01} + \mu_1^{02}) dt} \\
\text{Consider:} & \\
n P_x^{12} & \\
\implies n P_x^{12} &= S_0 t P_x^{12} \cdot \mu_1^{12} \cdot \Delta t \\
\text{For PDM} & \quad t P_x^{12} = t P_x^{12} \\
\text{Consider:} & \\
n P_x^{01} & \\
\implies n P_x^{01} &= S_0 t P_x^{00} \cdot \mu_1^{01} \cdot \Delta t \cdot n \cdot t P_x^{12} \\
\text{For PDM} & \quad t P_x^{00} = t P_x^{00} \\
& \quad n \cdot t P_x^{12} = n \cdot t P_x^{12} 
\end{align*}
\]
Consider:

\[ nP_x^{02} \] (easy way)

\[ nP_x^{02} = 1 - nP_x^{00} - nP_x^{01} \]

(harder way: 2-paths \( (0) \rightarrow (2) \) or \( (0) \rightarrow (1) \rightarrow (2) \))

\[ nP_x^{02} = I_1 + I_2 \]

\[ I_1: \]

\[ \int \]

\[ x^0 \]

\[ \Rightarrow \]

\[ x^2 \]

\[ t \]

\[ P_r = tP_x^{00} \cdot \lambda_{x+2} \cdot At \]

\[ I_1 = \sum_0^n tP_x^{00} \cdot \lambda_{x+2} \cdot rt \]

\[ I_2: \]

\[ \int \]

\[ x^0 \]

\[ \Rightarrow \]

\[ x^2 \]

\[ t \]

\[ P_r = tP_x^{00} \cdot \lambda_{x+2} \cdot At \]

\[ t < u < \hat{t} \]

\[ P_r = tP_x^{00} \cdot \lambda_{x+2} \cdot At \]

\[ I_2 = \sum_0^n tP_x^{00} \cdot \lambda_{x+2} \cdot \int_0^t u \cdot P_x^{00} \cdot \lambda_{x+2} \cdot du \cdot rt \]

If transitions between states are allowed, we can not determine exact values of these probabilities. Instead, we use Euler's method to approximate them.

Euler's Method (EM)

Recall the definition of the derivative of a function:
\[ y(t) = \lim_{h \to 0} \frac{\gamma(t+h) - \gamma(t)}{h} \quad \text{for small } h \]

Rewrite: \[ \gamma(t+h) = \gamma(t) + h \cdot \dot{\gamma}(t) \quad (\text{EM}) \]

- \( h \) = step size
- \( h > 0 \) \implies \text{"Forward Equation"} \nonumber
- \( h < 0 \) \implies \text{"Backward Equation"} \nonumber

For now, we'll be using \[ \gamma(t) = t \cdot \sigma \]

Note: \[ \sigma = \begin{cases} 0 & \text{if } j \neq i \\ 1 & \text{if } j = i \end{cases} \quad (\because \gamma(0) \text{ is known}) \]

? \( \gamma(0.1) = t \cdot \sigma \) is unknown.

We can approximate it using EM with stepsize \( h = 0.1 \) and initial condition \( \gamma(0) \rightarrow \text{known} \)

EM: \[ \gamma(t+h) = \gamma(t) + h \cdot \dot{\gamma}(t) \]

\( t=0 \)

\( h = 1 \)

\[ \sigma = \begin{cases} 0 \cdot \sigma & \text{known} \\ t \cdot \sigma & \text{known, later} \end{cases} \]

Once we know how to get \( t \cdot \sigma \), we'll be done.

Kolmogorov's Differential Equations (KDE's)

**Example:**

\[ (0) \rightarrow (1) \]

\( (2) \rightarrow (3) \)

\[ M \& S \]

? \( t \cdot \sigma \) = "rate of transitions into state 0 at time \( t \)"

\( t \cdot \sigma \) = "rate of transitions out of state 0 at time \( t \)"

\( \text{for an (x) in state 0} \)