MLC153 (Continued) Common Mortality Models

Constant Force Model $CF(\mu)$

$\mu_x = \mu$

For this model $X = T_0 \sim \text{Exp} \left( \text{mean} = \frac{1}{\mu} \right)$

Fact: $T_x = X - x \mid X > x \sim \text{Exp} \left( \text{mean} = \frac{1}{\mu} \right)$

$tP_x = e^{-\mu t} \quad \rightarrow \quad p_x = e^{-\mu} = p \quad ; \quad tP_x = (e^{-\mu})^t = p^t$

$
\begin{align*}
t\delta_x &= 1 - e^{-\mu t} \\
\delta_x(t) &= \mu e^{-\mu t}
\end{align*}$

MLC153: Idea: We have $l_x$ values for integer ages from an ILT. Want $l_x$ values for all $x$.

2 cases for extending discrete values to a continuous framework.
Case 1: Uniform Distribution of Deaths (UPD)

\[ x \text{-integer} \]
\[ 0 \leq t \leq 1 \]

\[
\text{slope } m = \frac{L_{x+1} - L_x}{(x+1) - x} = L_{x+1} - L_x
\]

equation of line segment:

\[ L_{x+t} - L_x = (L_{x+1} - L_x) \cdot t \]

Rewrite:  \[ L_{x+t} \overset{\text{UPD}}{=} (1-t) \cdot L_x + t \cdot L_{x+1}, \quad 0 \leq t \leq 1 \]

Fact:  \[ L_{x+t} \overset{\text{UPD (arithmetic)}}{=} \text{weighted average of } L_x \text{ and } L_{x+1} \]

E.g.  \[ L_{1.7} \overset{\text{UPD}}{=} 0.3 \cdot L_1 + 0.7 \cdot L_2 \]
Case 2: Constant Force (between integer ages)

\[ CF \implies tP_x = P = \frac{l_{x+t}}{l_x} \]

\[ \begin{align*}
  l_{x+t} &= l_x \cdot P^t \\
  p &= \frac{l_{x+t}}{l_x}
\end{align*} \]

\[ \therefore l_{x+t} = l_x \cdot \left( \frac{l_{x+t}}{l_x} \right)^t \]

\[ \therefore l_{x+t} = \frac{CF}{l_x} \cdot l_x^t \cdot \frac{1-t}{1-t} \quad 0 \leq t \leq 1 \]

Fact. \quad l_{x+t} \overset{CF}{=} \text{geometric weighted average of } l_x \text{ and } l_{x+t}

E.g. \quad l_{23.4} \overset{CF}{=} l_{23} \cdot l_{24}
Local vs Global Mortality

Case 1: UDD vs DML(w)

Case 2: CF lokal vs CF global

\[ y = l_x \]

\[ y = tP_x = e^{-rt} \]
Test 3 Review

Examples:

1) See #19, #20, #21(b) (discrete mixtures)

2) See #21(c) (continuous mixture)

Also see ML6M153 Exercises

Also this example: Determine the 2-year force of mortality that is consistent with the ILT centered at age 30. (from ages 29 to 31)

Solution: 

\[ 2p_{29} = \frac{l_{31}}{l_{29}} = e^{-2\mu} \]

\[ \Rightarrow \mu = \ldots \]

\[ \frac{9}{113} \cdot \frac{b_{38}}{l_{38}} = \frac{l_{42} - l_{45}}{l_{38}} \]