

1) (See Video Solution)

$${}_{30}P_{40:45} = .716580$$

2) mortality for (40) follows DML(120)  
 \_\_\_\_\_ (45) \_\_\_\_\_ CF ( $\mu = .025$ )

$$\begin{aligned} (a) \quad {}_{10|30}\overline{q}_{40:45} &= {}_{40}\overline{q}_{40:45} - {}_{30}\overline{q}_{40:45} \\ &= {}_{40}q_{40} \cdot {}_{40}q_{45} - {}_{30}q_{40} \cdot {}_{30}q_{45} \\ &= \frac{40}{120-40} \cdot (1 - e^{-40 \cdot (.025)}) - \frac{30}{120-40} \cdot (1 - e^{-30 \cdot (.025)}) = .118198 \end{aligned}$$

$$\begin{aligned} (b) \quad {}_{10}P_{40:45} &= 1 - {}_{10}\overline{q}_{40:45} = 1 - {}_{10}q_{40} \cdot {}_{10}q_{45} \\ &= 1 - \frac{10}{120-40} \cdot (1 - e^{-10 \cdot (.025)}) = .972350 \end{aligned}$$

$$\begin{aligned} (c) \quad {}_{30}\overline{q}_{50:55} &= {}_{30}q_{50} \cdot {}_{30}q_{55} = \frac{30}{120-50} \cdot (1 - e^{-30 \cdot (.025)}) \\ &= .226129 \end{aligned}$$

(d)  $.118198 \stackrel{?}{=} (.972350)(.226129)$  No

Generally  ${}_k|n\overline{q}_{xy} \neq {}_kP_{xy} \cdot n\overline{q}_{x+k:y+k}$  since  ${}_kP_{xy}$  does not imply that both (x) and (y) survive to ages  $x+k$  and  $y+k$ . We can not discuss  $n\overline{q}_{x+k:y+k}$  unless we are sure to have  $x+k$  and  $y+k$  year olds.

2)

$$e) {}_tP_{40:45} = {}_tP_{40} \cdot {}_tP_{45}$$

$$\text{For } t < 80, {}_tP_{40} = \frac{120-40-t}{120-40} = \frac{80-t}{80}$$

$${}_tP_{45} = e^{-.025t}$$

$$\therefore \text{for } t < 80, {}_tP_{40:45} = \frac{80-t}{80} \cdot e^{-.025t}$$

$$(f) \text{ For } t > 80, {}_tP_{40} = 0 \text{ since } \omega = 120.$$

$$\therefore \text{for } t > 80, {}_tP_{40:45} = {}_tP_{40} \cdot {}_tP_{45} = 0$$

$$(g) \ddot{e}_{40:45} = \int_0^{\infty} {}_tP_{40:45} dt = \int_0^{80} \frac{80-t}{80} \cdot e^{-.025t} dt$$

$$u = \frac{80-t}{80}$$

$$du = -\frac{1}{80} dt$$

$$v = -40e^{-.025t}$$

$$dv = e^{-.025t} dt$$

$$\therefore \ddot{e}_{40:45} = \left( \frac{80-t}{80} \right) (-40e^{-.025t}) \Big|_0^{80} - \int_0^{80} (-40e^{-.025t}) \cdot \frac{-1}{80} dt$$

$$= \underline{40} - \frac{1}{2} \int_0^{80} e^{-.025t} dt$$

$$= 40 - \frac{1}{2} \cdot \frac{1}{.025} e^{-.025t} \Big|_0^{80} = 40 - 20(1 - e^{-2})$$

$$= 20 + 20e^{-2} = 22.706706$$

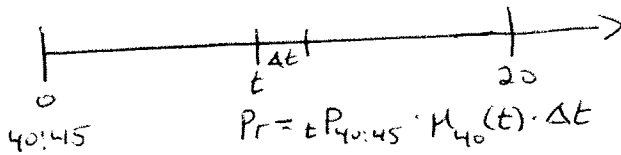
$$(h) \ddot{e}_{\overline{40:45}} : (\text{See Video Solution})$$

$$\ddot{e}_{\overline{40:45}} = 60 - 20e^{-2} = 57.293294$$

2)

$$(i) \quad {}_{20}q_{40:45} = 1 - {}_{20}P_{40:45} = 1 - {}_{20}P_{40} \cdot {}_{20}P_{45}$$

$$= 1 - \left( \frac{120-40-20}{120-40} \right) \cdot e^{-20(0.025)} = .545102$$

$$(j) \quad {}_{20}\bar{b}_{40:45} :$$


$$\therefore {}_{20}\bar{b}_{40:45} = \int_0^{20} {}_tP_{40:45} \cdot \mu_{40}(t) dt = \int_0^{20} \underline{{}_tP_{40}} \cdot \underline{{}_tP_{45}} \cdot \underline{\mu_{40}(t)} dt$$

Note: mortality for (40) follows DML(120)

$$\Rightarrow {}_tP_{40} \cdot \mu_{40}(t) = q_{40} = \frac{1}{120-40} = \frac{1}{80}$$

$$\therefore {}_{20}\bar{b}_{40:45} = \int_0^{20} \frac{1}{80} \cdot {}_tP_{45} dt = \frac{1}{80} \int_0^{20} e^{-0.025t} dt$$

$$= \frac{1}{80} \cdot \frac{1}{-0.025} e^{-0.025t} \Big|_0^{20} = \frac{1}{2} (1 - e^{-0.5}) = .196735$$

(k)  ${}_{20}\bar{b}_{40:45} :$  (See Video Solution)

$${}_{20}\bar{b}_{40:45} = \frac{1}{2} - \frac{1}{4} e^{-.5} = .348367$$

2)

$$(l) \quad {}_{10}q_{40:45} = {}_{10}q_{40} \cdot {}_{10}q_{45} = \frac{10}{120-40} \cdot (1 - e^{-0.25(10)})$$

$$= \frac{1}{8} (1 - e^{-.25}) = .027650$$

(m)  ${}_{10}q_{40:45}^2$  :

$Pr = {}_tP_{45} \cdot {}_tP_{40} \cdot \mu_{40}(t) \cdot \Delta t$

$$\therefore {}_{10}q_{40:45}^2 = \int_0^{10} {}_tP_{45} \cdot {}_tP_{40} \cdot \mu_{40}(t) dt$$

Note: mortality for (40) follows DML (120)

$$\Rightarrow {}_tP_{40} \cdot \mu_{40}(t) = q_{40} = \frac{1}{120-40} = \frac{1}{80}$$

$$\therefore {}_{10}q_{40:45}^2 = \int_0^{10} (1 - e^{-0.25t}) \cdot \frac{1}{80} dt$$

$$= \frac{1}{80} [10 + 40e^{-.025t}]_0^{10} = \frac{1}{80} [40e^{-.25} - 30]$$

$$= \frac{1}{2} e^{-.25} - \frac{3}{8} = .014400$$

(n)  ${}_{10}q_{40:45}^2$  : (See Video Solution)

$${}_{10}q_{40:45}^2 = \frac{1}{2} - \frac{5}{8} e^{-.25} = .013250$$

(o) ~~(See Video)~~

$${}_{30}q_{40:45} - {}_{30}q_{40:45}^2 = {}_{30}q_{40} \cdot {}_{30}P_{45} = \frac{30}{120-40} \cdot e^{-30(.025)}$$

$$= \frac{3}{8} e^{-.75} = .177137$$

$$3) \quad \mu = \mu^S \quad \mu^{NS} = \frac{1}{2} \mu$$

$$\begin{aligned} (a) \quad {}_{20|20}q_{30:50} &= {}_{20}P_{30:50} - {}_{40}P_{30:50} \\ &= {}_{20}P_{30} \cdot {}_{20}P_{50} - {}_{40}P_{30} \cdot {}_{40}P_{50} \\ &= \underline{e^{-20(\frac{1}{2}\mu)}} \cdot \underline{e^{-20(\mu)}} - \underline{e^{-40(\frac{1}{2}\mu)}} \cdot \underline{e^{-40(\mu)}} \end{aligned}$$

$$\therefore .09 = \underline{e^{-30\mu}} - \underline{e^{-60\mu}} \quad (\text{quadratic in } e^{-30\mu})$$

$$\begin{aligned} a &= -1 \\ b &= 1 \\ c &= -.09 \end{aligned} \quad \therefore e^{-30\mu} = \frac{-1 \pm \sqrt{1 - 4(-1)(-.09)}}{2(-1)} = \frac{+1 + \sqrt{.64}}{+2} = .9$$

$$\implies \mu = \frac{\ln(.9)}{-30} = .003512$$

(b) As in (a),

$$\begin{aligned} {}_{20|20}q_{30:50} &= {}_{20}P_{30} \cdot {}_{20}P_{50} - {}_{40}P_{30} \cdot {}_{40}P_{50} \\ &= e^{-20(\mu)} \cdot e^{-20(\frac{1}{2}\mu)} - e^{-40(\mu)} \cdot e^{-40(\frac{1}{2}\mu)} \\ &= e^{-30\mu} - e^{-60\mu} \end{aligned}$$

We get the same equation and the same answer as in part (a). Note that this won't always be the case; it just so happened in this problem.

4)

$$(a) {}_tP_{xy} = {}_tP_x \cdot {}_tP_y = e^{-\mu t} \cdot e^{-\mu t} = e^{-2\mu t} \Rightarrow T_{xy} \sim \text{Exp}(\text{mean} = \frac{1}{2\mu})$$

$$(b) \text{ Using CF}(2\mu) \text{ mortality, } {}_tP_x = e^{-2\mu t} \text{ also.}$$

$$(c) \dot{e}_{xy} = \frac{1}{2\mu} \text{ since } T_{xy} \sim \text{Exp}(\text{mean} = \frac{1}{2\mu})$$

$$(d) \dot{e}_{\overline{xy}} = \dot{e}_x + \dot{e}_y - \dot{e}_{xy} = \frac{1}{\mu} + \frac{1}{\mu} - \frac{1}{2\mu} = \frac{2+2-1}{2\mu} = \frac{3}{2\mu}$$

5) (See Video Solution)

$$(a) {}_tP_{xx} = \left(\frac{w-x-t}{w-x}\right)^2$$

$$(b) {}_tP_x \stackrel{\text{GDML}(w,2)}{=} \left(\frac{w-x-t}{w-x}\right)^2$$

$$(c) \dot{e}_{xx} = \frac{w-x}{3}$$

$$(d) \dot{e}_{\overline{xx}} = \frac{2(w-x)}{3}$$

$$(b) (a) {}_tP_{70:70} = ({}_tP_{70})^2 = \left(\frac{100-70-t}{100-70}\right)^2 = \left(\frac{30-t}{30}\right)^2 \text{ Note } 0 \leq t \leq 30$$

$$(b) \dot{e}_{70:70} = \int_0^{30} \left(\frac{30-t}{30}\right)^2 dt = \frac{30}{3} \left(\frac{30-t}{30}\right)^3 \Big|_0^{30} = 10$$

$$\text{Note: (See \#5(c)) } \dot{e}_{70:70} \stackrel{\text{PML}(100)}{=} \frac{100-70}{3} = \frac{30}{3} = 10$$

$$(c) \dot{e}_{70:70:\overline{20}|} = \int_0^{20} \left(\frac{30-t}{30}\right)^2 dt = \frac{30}{3} \left(\frac{30-t}{30}\right)^3 \Big|_0^{20}$$

$$= 10(1)^3 - 10\left(\frac{1}{3}\right)^3 = 10\left(1 - \frac{1}{27}\right) = \frac{260}{27}$$

$$7) \mu_x^* = .02 \quad \mu_y^* = .04 \quad \lambda = .01 \Rightarrow \mu_x = .03 \quad \mu_y = .05 \quad \mu_{xy} = .07$$

$$(a) {}_{10}q_x = 1 - {}_{10}p_x \stackrel{CF}{=} 1 - e^{-10(.03)} = 1 - e^{-.3}$$

$$(b) \text{ expected age when } (y) \text{ dies} = y + E[T_y] = y + \dot{e}_y$$

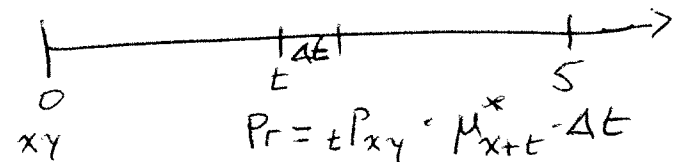
$$\dot{e}_y \stackrel{CF}{=} \frac{1}{\mu_y} = \frac{1}{.05} = 20$$

$$\therefore \text{ expected age when } (y) \text{ dies} = y + 20$$

$$8) \text{ Given } \underbrace{E[T_x^*]}_{= \frac{1}{\mu_x^*}} = \underbrace{E[T_y]}_{= \frac{1}{\mu_y} = \frac{1}{\mu_y^* + \lambda}} \Rightarrow \mu_x^* = \underline{\mu_y^* + \lambda}$$

$$.2 = \mu_{xy} = \mu_x^* + \underline{\mu_y^* + \lambda} = 2\mu_x^* \Rightarrow \mu_x^* = .1$$

We seek  ${}_5q_{xy}^*$  :



$$Pr = {}_tP_{xy} \cdot \mu_{x+t}^* \cdot \Delta t$$

↳ We use  $\mu^*$  and not  $\mu$  since we seek the probability that (x) dies before (y)

$$\therefore {}_5q_{xy}^* = \int_0^5 {}_tP_{xy} \cdot \mu_{x+t}^* dt = \int_0^5 e^{-.2t} (.1) dt$$

$$= \frac{.1}{.2} e^{-.2t} \Big|_0^5 = \frac{1}{2} (1 - e^{-1})$$