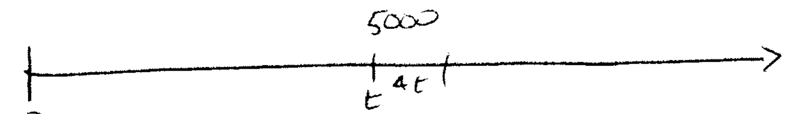
1) $\bar{A}_{x:\overline{n|}} = \bar{A}^{1}_{x:\overline{n|}} + A_{x:\overline{n|}}^{\ \ 1} = \bar{A}^{1}_{x:\overline{n|}} + {}_nE_x$

$\bar{A}^{1}_{x:\overline{n|}} = \bar{A}_x - {}_nE_x \cdot \bar{A}_{x+n}$

$= .5 - (.55)(.6) = .17$

$\therefore \bar{A}_{x:\overline{n|}} = .17 + .55 = .72$

2) (See Video Solution)



0

60

$PVB = 5000\,v^t$

$Pr = {}_tp_{60} \cdot \mu_{60+t} \cdot \Delta t$

$PVRV = Z = 5000\,\bar{Z}_{60} = 5000\,v^T \qquad T = T_{60}$

(a) DML(100) mortality and $\delta = .05$

$EPV = E[Z] = 5000\,\bar{A}_{60} = 2161.66$

$Var(Z) = 5000^2\left[{}^2\bar{A}_{60} - (\bar{A}_{60})^2\right] = 1{,}462{,}745$

(b) CF ($\mu = .03$, $\delta = .05$)

$EPV = E[Z] = 5000\,\bar{A}_{60} = 1875$

$Var(Z) = 5000^2\left[{}^2\bar{A}_{60} - (\bar{A}_{60})^2\right] = 2{,}253{,}605$

3)

$$Z = PVRV = 1000\ {}_{20|}\bar{Z}_{60} = \begin{cases} 0 & \text{if } T \le 20 \\ 1000 v^T & \text{if } T > 20 \end{cases} \qquad T = T_{60}$$

(a) DML(100) mortality and $\delta = .05$

$$EPV = E[Z] = 1000\ {}_{20|}\bar{A}_{60} = 1000\ {}_{20}E_{60} \cdot \bar{A}_{80}$$

$${}_{20}E_{60} = v^{20} \cdot {}_{20}p_{60} = e^{-20(.05)}\ \frac{100-60-20}{100-60} = .18394$$

$$\bar{A}_{80} \overset{DML(100)}{=\!=\!=} \frac{1}{20}\,\bar{a}_{\overline{20|}} = \frac{1}{20} \cdot \frac{1-v^{20}}{\delta} = \frac{1}{20} \cdot \frac{1-e^{-20(.05)}}{.05} = .63212$$

$$\therefore EPV = 1000(.18394)(.63212) = 116.27$$

$$Var(Z) = 1000^2\left[{}^2_{20|}\bar{A}_{60} - ({}_{20|}\bar{A}_{60})^2\right]$$

$${}_{20|}\bar{A}_{60} = \underline{{}_{20}E_{60}} \cdot \underline{\bar{A}_{80}} = \underline{v^{20} \cdot {}_{20}p_{60}} \cdot \underline{\frac{1}{20} \cdot \frac{1-v^{20}}{\delta}} = .11627$$

$${}^2_{20|}\bar{A}_{60} = \underline{{}^2_{20}E_{60}} \cdot \underline{{}^2\bar{A}_{80}} = \underline{v^{40} \cdot {}_{20}p_{60}} \cdot \underline{\frac{1}{20}\ \frac{1-v^{40}}{2\delta}} = .02925$$

$$= 1000^2\left[.02925 - (.11627)^2\right] = 15730$$

(b) $CF(\mu = .03,\ \delta = .05)$

$$EPV = E[Z] = 1000\ {}_{20|}\bar{A}_{60} = 1000\ {}_{20}E_{60} \cdot \bar{A}_{80}$$

$${}_{20}E_{60} = v^{20} \cdot {}_{20}p_{60} = e^{-20\delta} \cdot e^{-20\mu} = e^{-20(\mu+\delta)} = .2019$$

$$\bar{A}_{80} \overset{CF}{=\!=} \frac{\mu}{\mu+\delta} = \frac{3}{8} = .375$$

$$\therefore EPV = 1000(.2019)(.375) = 75.71$$

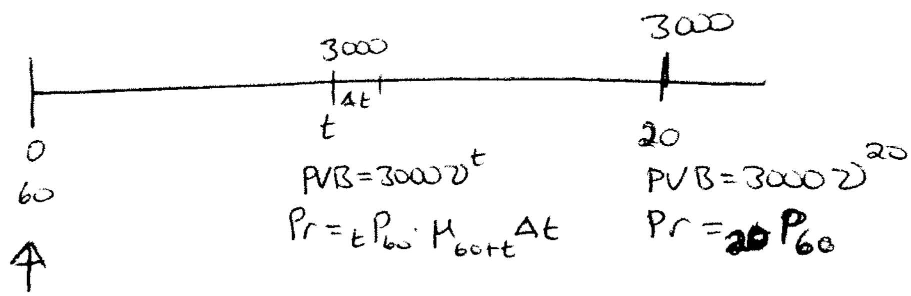
$$Var(Z) = 1000^2\left[{}^2_{20|}\bar{A}_{60} - ({}_{20|}\bar{A}_{60})^2\right]$$

$${}_{20|}\bar{A}_{60} = {}_{20}E_{60} \cdot \bar{A}_{80} = v^{20} \cdot {}_{20}p_{60} \cdot \frac{\mu}{\mu+\delta} = e^{-20(\mu+\delta)} \cdot \frac{\mu}{\mu+\delta} = .07571$$

$${}^2_{20|}\bar{A}_{60} = e^{-20(\mu+2\delta)}\ \frac{\mu}{\mu+2\delta} = .01714$$

$$= 1000^2\left[.01714 - (.07571)^2\right] = 11410$$

4) (See Video Solution)



$$PVRV = Z = 3000\,\bar{Z}_{60:\overline{20|}} = \begin{cases} 3000v^T & \text{if } T \le 20 \\ 3000v^{20} & \text{if } T > 20 \end{cases} \qquad T = T_{60}$$

(a) DML(100) mortality and $\delta = .05$

$$EPV = E[Z] = 3000\,\bar{A}_{60:\overline{20|}} = 1500$$

$$Var(Z) = 3000^2\left[{}^2\bar{A}_{60:\overline{20|}} - (\bar{A}_{60:\overline{20|}})^2\right] = 304470$$

(b) CF($\mu = .03$, $\delta = .05$)

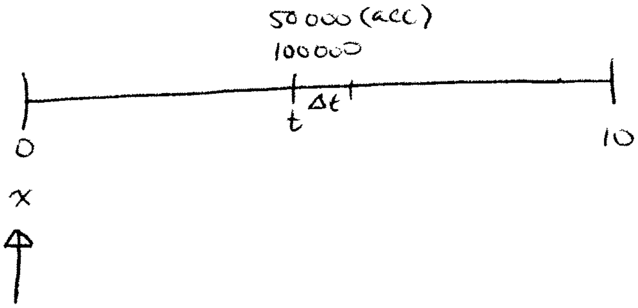
$$EPV = E[Z] = 3000\,\bar{A}_{60:\overline{20|}} = 1503.56$$

$$Var(Z) = 3000^2\left[{}^2\bar{A}_{60:\overline{20|}} - (\bar{A}_{60:\overline{20|}})^2\right] = 330,400$$

5) (See Video Solution)

$$APV = 10\bar{A}_x + 20\bar{A}_y - 5\bar{A}_{xy}$$

6)



$$EPV = 100000\,\bar{A}^{\,1}_{x:\overline{10|}} + 50000\,\bar{A}^{1\,(acc)}_{x:\overline{10|}}$$

$$\bar{A}^{\,1}_{x:\overline{10|}} = \bar{A}_x - {}_{10}E_x \cdot \bar{A}_{x+10} \overset{CF}{=} \frac{\mu^{(\tau)}}{\mu^{(\tau)}+\delta} - e^{-10(\mu^{(\tau)}+\delta)} \cdot \frac{\mu^{(\tau)}}{\mu^{(\tau)}+\delta}$$

$$= \frac{.025}{.075} - e^{-10(.075)} \cdot \frac{.025}{.075} = .17588$$

$$\bar{A}^{1\,(acc)}_{x:\overline{10|}} = \int_0^{10} v^t \cdot {}_tp_x \cdot \mu^{(acc)}_{x+t}\,dt$$

$$\mu^{(acc)}_{x+t} = .005\,, \quad \mu^{(\tau)}_{x+t} = .025 \implies \mu^{(acc)}_{x+t} = \frac{1}{5}\mu^{(\tau)}_{x+t}$$

$$\therefore \bar{A}^{1\,(acc)}_{x:\overline{10|}} = \frac{1}{5}\int_0^{10} v^t \cdot {}_tp_x \cdot \mu^{(\tau)}_{x+t}\,dt = \frac{1}{5} A^{\,1}_{x:\overline{10|}}$$

$$= \frac{1}{5}(.17588) = .03518$$

$$\therefore EPV = 100000(.17588) + 50000(.03518)$$

$$= 19347$$

7) $EPV = 1000\,\bar{A}_{40}$

(a) $\bar{A}_{40} \overset{UDD}{=} \frac{i}{\delta} A_{40} \overset{ILT}{=} \frac{.06}{\ln(1.06)}(.16132)$

$\Longrightarrow EPV = 1000\,\frac{.06}{\ln(1.06)}(.16132) = 166.11$

(b) $\bar{A}_{40} \overset{CAA}{=} (1+i)^{1/2} A_{40} \overset{ILT}{=} (1.06)^{1/2}(.16132)$

$\Longrightarrow EPV = 1000\,(1.06)^{1/2}(.16132) = 166.09$

8) $EPV = 1000\,\bar{A}_{40:\overline{20|}} = 1000\,\bar{A}^{1}_{40:\overline{20|}} + \underbrace{1000\,{}_{20}E_{40}}_{\stackrel{ILT}{=} 274.14}$

(a) $\bar{A}^{1}_{40:\overline{20|}} \stackrel{UDD}{=} \frac{i}{\delta} A^{1}_{40:\overline{20|}}$

$= \frac{i}{\delta}\left[A_{40} - {}_{20}E_{40} \cdot A_{60} \right]$

$\stackrel{ILT}{=} \frac{.06}{\ln(1.06)}\left[.16132 - (.27414)(.36913) \right] = .06191$

$\therefore EPV = 1000(.06191) + 274.14 = 336.05$

(b) $\bar{A}^{1}_{40:\overline{20|}} \stackrel{CAA}{=} (1+i)^{1/2} A^{1}_{40:\overline{20|}}$

$\underset{\text{as above}}{\stackrel{ILT}{=}} (1.06)^{1/2}\left[.16132 - (.27414)(.36913) \right]$

$= .06190$

$\therefore EPV = 1000(.06190) + 274.15 = 336.04$