Solutions to MLCM354 Exercises

1) $$P_{40} = \frac{A_{40}}{\ddot{a}_{40}} \overset{ILT}{=} \frac{.16132}{14.8166} \doteq .010888$$

2) $$P^{1}_{40:\overline{20}|} = \frac{A^{1}_{40:\overline{20}|}}{\ddot{a}_{40:\overline{20}|}}$$

$$A^{1}_{40:\overline{20}|} = A_{40} - {}_{20}E_{40} \cdot A_{60} \overset{ILT}{=} .060127$$

$$\ddot{a}_{40:\overline{20}|} = \ddot{a}_{40} - {}_{20}E_{40} \cdot \ddot{a}_{60} \overset{ILT}{=} 11.7612$$

$$\therefore P^{1}_{40:\overline{20}|} \overset{ILT}{=} .005112$$

3) $$P_{40:\overset{1}{\overline{20}|}} = \frac{A_{40:\overset{1}{\overline{20}|}}}{\ddot{a}_{40:\overline{20}|}} = \frac{{}_{20}E_{40}}{\ddot{a}_{40:\overline{20}|}} \overset{ILT}{=} \frac{.27414}{11.7612} = .023309$$

4) $$P_{40:\overline{20}|} = \frac{A_{40:\overline{20}|}}{\ddot{a}_{40:\overline{20}|}} = \frac{A^{1}_{40:\overline{20}|} + A_{40:\overset{1}{\overline{20}|}}}{\ddot{a}_{40:\overline{20}|}} = P^{1}_{40:\overline{20}|} + P_{40:\overset{1}{\overline{20}|}}$$

$$= .005112 + .023309 = .028421$$

OR, since $\ddot{a}_{40:\overline{20}|} = \dfrac{1 - A_{40:\overline{20}|}}{d}$,

$$P_{40:\overline{20}|} = \frac{1 - d\,\ddot{a}_{40:\overline{20}|}}{\ddot{a}_{40:\overline{20}|}} = \frac{d \cdot A_{40:\overline{20}|}}{1 - A_{40:\overline{20}|}}$$

5) $$P_{40:50} = \frac{A_{40:50}}{\ddot{a}_{40:50}} \stackrel{\text{ILT}}{=} \frac{.29368}{12.4784} = .023535$$

6) $$P_{\overline{40:50}} = \frac{A_{\overline{40:50}}}{\ddot{a}_{\overline{40:50}}} = \frac{A_{40} + A_{50} - A_{40:50}}{\ddot{a}_{40} + \ddot{a}_{50} - \ddot{a}_{40:50}}$$

$$\stackrel{\text{ILT}}{=} \frac{.16132 + .24905 - .29368}{14.8166 + 13.2668 - 12.4784} = .007478$$

Note that, like with force of mortality, with premiums

$$P_{\overline{xy}} \neq P_x + P_y - P_{xy}$$

7) $\pi \cdot \ddot{a}_{40:\overline{20}|} = 5000 A_{40}$

$$\Rightarrow \pi = \frac{5000 A_{40}}{\ddot{a}_{40:\overline{20}|}} \underset{\text{See \#2}}{\overset{\text{ILT}}{=\!=}} \frac{5000(.16132)}{11.7612} = 68.58$$

8) $\pi \cdot \ddot{a}_{40} = 5000 \bar{A}_{40}$

$$\Rightarrow \pi = \frac{5000 \bar{A}_{40}}{\ddot{a}_{40}} \underset{\text{ILT}}{\overset{\text{CAA}}{=\!=}} \frac{5000(1.06)^{1/2}(.16132)}{14.8166} = 56.05$$

9) $4\pi \cdot \ddot{a}_{40}^{(4)} = 5000 \bar{A}_{40}$

$$\Rightarrow \pi = \frac{5000 \bar{A}_{40}}{4 \cdot \ddot{a}_{40}^{(4)}} \underset{\text{ILT}}{\overset{\text{UDD}}{=\!=}} \frac{5000\left(\frac{.06}{\ln(1.06)}\right)(.16132)}{4[1.00027(14.8166) - .38424]} = 14.38$$

Note: $\bar{A}_{40} \overset{\text{UDD}}{=\!=} \frac{i}{\delta} \cdot A_{40}$

$\ddot{a}_{40}^{(4)} \overset{\text{UDD}}{=\!=} \alpha(4) \cdot \ddot{a}_{40} - \beta(4)$

10) $12\pi\, \ddot{a}^{(12)}_{40:\overline{20|}} = 10000\, A^{\,1}_{40:\overline{20|}}$

$$\Rightarrow \pi = \frac{10000\, A^{\,1}_{40:\overline{20|}}}{12 \cdot \ddot{a}^{(12)}_{40:\overline{20|}}}$$

$$A^{\,1}_{40:\overline{20|}} = A_{40} - {}_{20}E_{40} \cdot A_{60} \overset{ILT}{=} .060127$$

$$\ddot{a}^{(12)}_{40:\overline{20|}} = \ddot{a}^{(12)}_{40} - {}_{20}E_{40} \cdot \ddot{a}^{(12)}_{60}$$

$$\ddot{a}^{(12)}_{40} \overset{WH}{=} \ddot{a}_{40} - \frac{11}{24} - \frac{143}{1728}(\mu_{40} + \delta)$$

$$\delta = \ln(1.06)$$

$${}_2P_{39} = e^{-2\mu_{40}} \Rightarrow \mu_{40} = -\frac{1}{2}\ln\left(\frac{\ell_{41}}{\ell_{39}}\right)$$

$$\therefore \ddot{a}^{(12)}_{40} \overset{WH}{\underset{ILT}{=}} 14.353222$$

$$\ddot{a}^{(12)}_{60} \overset{WH}{=} \ddot{a}_{60} - \frac{11}{24} - \frac{143}{1728}(\mu_{60} + \delta)$$

$$\mu_{60} = -\frac{1}{2}\ln\left(\frac{\ell_{61}}{\ell_{59}}\right)$$

$$\therefore \ddot{a}^{(12)}_{60} \overset{WH}{\underset{ILT}{=}} 10.681146$$

$$\therefore \ddot{a}^{(12)}_{40:\overline{20|}} \overset{WH}{\underset{ILT}{=}} 11.425092$$

$$\Rightarrow \pi = 4.39$$

11) (See Video Solution)

$Var({}_0L) = 32139$

12) Total Loss $= S = \sum_{1}^{100} ({}_0L)_i$

Standard Normal Distribution

We seek $Pr(S \le 1500) \approx Pr\left(SND \le \frac{1500 - E[S]}{\sqrt{Var(S)}}\right)$

${}_0L$ is as in #11

$E[S] = 100\, E[{}_0L] = 0$ since using premium from equivalence principle

$Var(S) = 100\, Var({}_0L)$ (See #11)

$\therefore\ Pr(S \le 1500) \approx Pr\left(SND \le \frac{1500 - 0}{\sqrt{100\, Var({}_0L)}} \approx .84\right) = .7995$

13) (See Video Solution)

$\pi = 10000\, P_{61} = 764$

14) $4\pi\, \ddot{a}_x^{(4)} = 1000\, A_x^{(4)}$ $\qquad A_x^{(4)} = 1 - d^{(4)} \cdot \ddot{a}_x^{(4)}$

$\Rightarrow \pi = \dfrac{1000\left(1 - d^{(4)}\, \ddot{a}_x^{(4)}\right)}{4\, \ddot{a}_x^{(4)}}$

$\left(1 - \frac{d^{(4)}}{4}\right)^4 = 1 - d = .9 \Rightarrow d^{(4)} = 4\left(1 - (.9)^{.25}\right)$

$\ddot{a}_x^{(4)} \underset{\text{2-term WH}}{=} \ddot{a}_x - \frac{3}{8} = 5.625$

$\therefore\ \pi = 18.45$