

# Solutions to MLCM357 Exercises

1) See Video Solution

$$P = .9772$$

$$2) S = \sum_1^{10000} ({}_0L)_i \quad {}_0L = 100000 Z_{60} - \pi \ddot{Y}_{60}$$

We seek  $\pi$  such that  $\Pr(S > 0) = .01$

$$E[S] = 10000 E[{}_0L]$$

$$E[{}_0L] = 100000 A_{60} - \pi \ddot{a}_{60} \stackrel{ILT}{=} 36913 - 11.1454 \pi$$

$$\text{Var}(S) = 10000 \text{Var}({}_0L) \quad \ddot{Y}_{60} = \frac{1 - Z_{60}}{d}$$

$$\cancel{\text{Var}} \quad {}_0L = (100000 + \frac{\pi}{d}) Z_{60} - \frac{\pi}{d}$$

$$\text{Var}({}_0L) = (100000 + \frac{\pi}{d})^2 [{}^2A_{60} - (A_{60})^2]$$

$$\therefore \text{Var}(S) = 10000 (100000 + \frac{\pi}{d})^2 [{}^2A_{60} - (A_{60})^2]$$

$$\Rightarrow \sqrt{\text{Var}(S)} = 100 (100000 + \frac{\pi}{d}) \sqrt{{}^2A_{60} - (A_{60})^2} \quad d = \frac{.06}{1.06}$$

$$\stackrel{ILT}{=} 2028621.283 + 358.38976 \pi$$

$$\therefore \Pr(S > 0) = \Pr(\text{SND} > \frac{-E[S]}{\sqrt{\text{Var}(S)}} = \frac{(11.1454\pi - 36913)(10000)}{2028621.283 + 358.38976\pi}) = .01$$

standard normal distribution

 $\Rightarrow$ 

 $= 99^{\text{th}}$  percentile of SND

$$\therefore \frac{10000(11.1454\pi - 36913)}{2028621.283 + 358.38976\pi} = 2.326 \Rightarrow \pi = 3379.56$$

$$3) \quad {}_0L = 5000 \bar{Z}_x - \pi \cdot \bar{Y}_x \quad \bar{Y}_x = \frac{1 - \bar{Z}_x}{\delta}$$

$$= \left(5000 + \frac{\pi}{\delta}\right) \bar{Z}_x - \frac{\pi}{\delta} \stackrel{\delta = .04}{=} (5000 + 25\pi) \bar{Z}_x - 25\pi$$

$$\therefore .95 = \Pr({}_0L < 0) = \Pr\left(\bar{Z}_x < \frac{25\pi}{5000 + 25\pi}\right)$$

$$\bar{Z}_x = e^{-\delta T} = e^{-.04T}$$

$$\Rightarrow .95 = \Pr\left(\bar{Z}_x < \frac{25\pi}{5000 + 25\pi}\right)$$

$$= \Pr\left(T > \frac{\ln\left(\frac{25\pi}{5000 + 25\pi}\right)}{-.04}\right)$$

$$= \Pr\left(T > \underbrace{-25 \ln\left(\frac{25\pi}{5000 + 25\pi}\right)}_{=t}\right)$$

$$= {}_tP_x = e^{-\mu t}$$

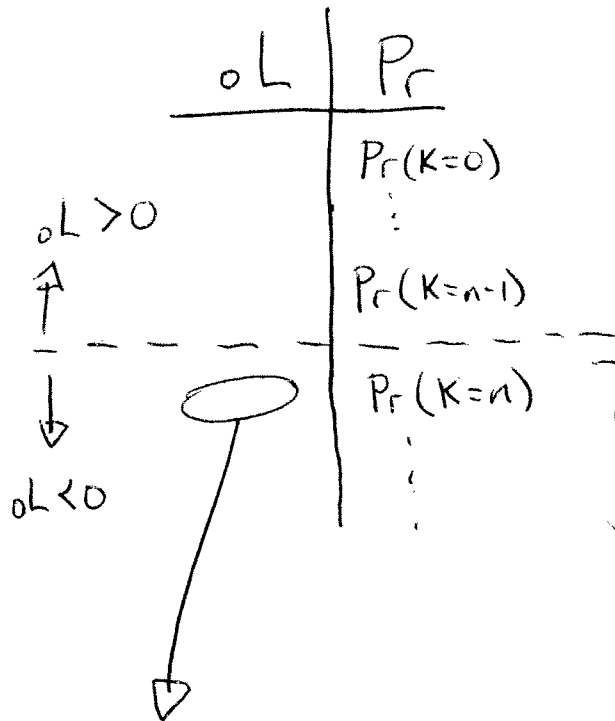
$$= e^{-.02 \left[-25 \ln\left(\frac{25\pi}{5000 + 25\pi}\right)\right]}$$

$$= e^{.5 \ln\left(\frac{25\pi}{5000 + 25\pi}\right)} = \left(\frac{25\pi}{5000 + 25\pi}\right)^{.5}$$

$$\therefore (.95)^2 = \frac{25\pi}{5000 + 25\pi} \Rightarrow \pi = 1851.28$$

$$4) \quad {}_0L = 5000 Z_{35} - \pi \ddot{Y}_{35}$$

$$= 5000 v^{k+1} - \pi \ddot{a}_{\overline{k+1}|}$$



$${}_0L = 5000 v^{n+1} - \pi \ddot{a}_{\overline{n+1}|}$$

$$\left. \begin{array}{l} Pr(k=0) \\ \vdots \\ Pr(k=n-1) \\ Pr(k=n) \\ \vdots \\ Pr(k=n) \end{array} \right\} \Sigma = {}_n P_{35} = \frac{l_{35+n}}{l_{35}} > .95$$

$$\Rightarrow l_{35+n} > .95 l_{35} \stackrel{ILT}{=} 8949624.15$$

$$l_{50} = 8950901 \quad l_{51} = 8897913$$

$$\therefore n = 15$$

$$5000 v_{.06}^{16} - \pi \ddot{a}_{\overline{16}|.06} < 0$$

$$\Rightarrow \pi \geq 183.74$$