MAP 4170 Name: KEY
Test 1 Date: September 15, 2020

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Account *A* credits interest using a simple interest rate of 5%. Account *B* credits interest using a simple discount rate, *d*. At time $t = 5$, the forces of interest in the two accounts are equal. If 1000 is deposited into account *B* at time $t = 0$, how much will be in the account at time $t = 5$.

(A) 1190
(B) 1195
(C) 1200
(D) 1205
(E) 1210

Account A	Account B
$i = 0.05$ simple	d - simple
$a(t) = 1 + 0.05t$	$a(t) = (1 - dt)^{-1}$
$\delta_t = \frac{0.05}{1+0.05t}$	$\delta_t = \frac{d}{1-dt}$
$\delta_5 = \frac{0.05}{1.25}$	$\delta_5 = \frac{d}{1-5d}$

$$\frac{0.05}{1.25} = \frac{d}{1-5d} \quad \Rightarrow \quad d = 0.0333\cdots$$

$$B\text{: } A(5) = 1000a(5) = 1000(1 - 5(0.0333\cdots))^{-1} = 1200$$

2. Olivia is to receive payments of *X* in 2 years and *Y* in 5 years. Aidan is to receive payments of $2X$ in 2 years and $5Y$ in 5 years. Using an annual effective interest rate of 6%, the present value of Olivia's payments is 460, and the present value of Aidan's payments is 2025. Determine *X*.

(A) 100
(B) 103
(C) 106
(D) 109
(E) 112

$$i = 0.06 \quad \Rightarrow \quad aaf = 1.06 \quad \Rightarrow \quad adf = \frac{1}{1.06} = v$$

$$(460 = Xv^2 + Yv^5) \cdot (-5)$$
$$2025 = 2Xv^2 + 5Yv^5$$

$$-2300 = -5Xv^2 - 5Yv^5$$
$$2025 = 2Xv^2 + 5Yv^5$$

$$-275 = -3Xv^2$$

$$X = \frac{275}{3}v^{-2} = \frac{275}{3}(1.06)^2 = 103$$

3. Two 180-day T-Bills, one Canadian and the other US, each have a quoted rate of 6% and each have a redemption value of 1000. If C is the price of the Canadian T-Bill and U is the price of the US T-Bill, determine the value of $U - C$.

(A) – 1.25

(B) – 0.50

(C) 0.25

(D) 1.00

(E) 1.75

$C\left(1 + 0.06 \cdot \frac{180}{365}\right) = 1000$

$\Rightarrow \quad C = 971.26$

$U(1 - 0.06 \cdot \frac{180}{360})^{-1} = 1000$

$\Rightarrow \quad U = 970$

$\therefore \quad U - C = -1.26$

4. Using the same nominal discount rate, d, compounded semiannually, a payment of 1000 at the end of 10 years has the same present value as a payment of 590 at the end of 5 years. Determine d.

(A) 4.56%

(B) 5.14%

(C) 7.81%

(D) 9.12%

(E) 10.28%

$d = d^{(2)} \quad \Rightarrow \quad \frac{d}{2} = sedr \quad \Rightarrow \quad 1 - \frac{d}{2} = sdf = v$

$1000v^{20} = 590v^{10} \quad \Rightarrow \quad v^{10} = 0.59$

$\therefore \quad (1 - \frac{d}{2})^{10} = 0.59 \quad \Rightarrow \quad d = 0.10279\cdots$

5. Given a simple interest rate of 4%, determine the ratio $\frac{d_4}{i_5}$ where d_4 is the annual effective discount rate for the fourth year, and i_5 is the annual effective interest rate for the fifth year.

(A) $\frac{112}{116}$

(B) $\frac{116}{120}$

(C) $\frac{120}{116}$

(D) $\frac{116}{112}$

(E) None of the above

$i = 0.04$ simple $\Rightarrow \quad a(t) = 1 + 0.04t$

$d_4 = \frac{a(4)-a(3)}{a(4)} = \frac{1.16-1.12}{1.16} = \frac{0.04}{1.16}$

$i_5 = \frac{a(5)-a(4)}{a(4)} = \frac{1.20-1.16}{1.16} = \frac{0.04}{1.16}$

$\therefore \quad \frac{d_4}{i_5} = 1$

6. An account credits interest using a force of interest $\delta_t = \frac{0.25t^{-0.5}}{1+t^{0.5}}$. A deposit at time $t = 0$ doubles after n years. Determine n.

(A) 6

(B) 7

(C) 8

(D) 9

(E) 10

$\delta_t = \frac{0.25t^{-0.5}}{1+t^{0.5}} = \frac{1}{2} \cdot \frac{0.5t^{-0.5}}{1+t^{0.5}}$

$f(t) = 1 + t^{0.5}$

$c = \frac{1}{2}$

$a(t) = \left(\frac{f(t)}{f(0)}\right)^c = (1 + t^{0.5})^{0.5} = \sqrt{1 + \sqrt{t}}$

$X \cdot a(n) = 2X \quad \Rightarrow \quad a(n) = 2$

$\Rightarrow \quad \sqrt{1 + \sqrt{n}} = 2$

$\Rightarrow \quad n = 9$

7. An account credits interest using a simple discount rate of 10% for the first year, an annual effective discount rate of 10% for the second year, and a force of interest of 10% thereafter. An amount X is deposited into the account at the beginning of the first year. The accumulated value of the deposit is 1000 at the end of the fourth year. Determine X.

(A) 660

(B) 670

(C) 680

(D) 690

(E) 700

(For year 1) $d = 0.1$ simple $\Rightarrow$ $a(t) = (1 - 0.1t)^{-1}$

(For year 2) $d = 0.1$ aedr $\Rightarrow$ $aaf = (0.9)^{-1}$

(Thereafter) $\delta = 0.1$ $\Rightarrow$ $aaf = e^{0.1}$

$1000 = X \cdot paf_0^4 = X \cdot paf_0^1 \cdot paf_1^2 \cdot paf_2^4$

$paf_0^1 = a(1) = (0.9)^{-1}$

$paf_1^2 = aaf = (0.9)^{-1}$

$paf_2^4 = (e^{0.1})^2 = e^{0.2}$

$\therefore \quad 1000 = X \cdot (0.9)^{-1} \cdot (0.9)^{-1} \cdot e^{0.2} \quad \Rightarrow \quad X = 663.17$

8. An account credits interest using simple interest rate, i. A deposit at time $t = 0$ accumulates to 300 after 2.5 years, and it accumulates to 350 after 5 years. Determine i.

(A) 6%

(B) 8%

(C) 10%

(D) 12%

(E) 14%

$i -$ simple $\Rightarrow$ $a(t) = 1 + it$

$350 = 300 \cdot paf_{2.5}^5 = 300 \cdot \frac{a(5)}{a(2.5)} = 300 \cdot \frac{1+5i}{1+2.5i}$

$\therefore \quad i = 0.08$

9. Using a nominal interest rate, $i^{(2)}$, the total present value of payments of 400 at the end of two years and 600 at the end of four years is 908.20. Determine $i^{(2)}$.

(A) 1.50%

(B) 1.52%

(C) 3.00%

(D) 3.05%

(E) 3.10%

$$\frac{i^{(2)}}{2} = seir \quad \Rightarrow \quad sdf = \left(1 + \frac{i^{(2)}}{2}\right)^{-1} = v$$

$$908.20 = 400v^4 + 600v^8$$

$$\text{quadratic in } v^4; a = 600, b = 400, c = -908.2$$

$$v^4 = \frac{-400 \pm \sqrt{400^2 - 4(600)(-908.2)}}{2(600)} = 0.94133\cdots$$

$$\therefore \quad \left(1 + \frac{i^{(2)}}{2}\right)^{-4} = 0.94133\cdots \quad \Rightarrow \quad i^{(2)} = 0.03045\cdots$$

10. At an annual effective interest rate of 8%, determine the equivalent monthly effective interest rate, *m*, and the equivalent bi-annual effective interest rate, *b*.

(A) m = 0.64% and b = 16%

(B) m = 0.67% and b = 16%

(C) m = 0.64% and b = 16.64%

(D) m = 0.67% and b = 16.64%

(E) m = 0.80% and b = 16%

$$i = 0.08\ aeir \quad \Rightarrow \quad aaf = 1.08$$

$$m = meir \quad \Rightarrow \quad maf = 1 + m$$

$$b = baeir \quad \Rightarrow \quad baaf = 1 + b$$

$$aaf = maf^{12} \quad \Rightarrow \quad 1.08 = (1 + m)^{12}$$
$$\Rightarrow \quad m = 0.00643\cdots$$

$$aaf^2 = baaf \quad \Rightarrow \quad 1.08^2 = 1 + b$$
$$\Rightarrow \quad b = 0.1664$$