MAP 4170
Test 1

Name: KEY
Date: September 16, 2021

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Two 180-day T-Bills, one Canadian and the other U.S., have the same price, and both have a quoted rate of 10%. The redemption value of the Canadian T-Bill is 1000. Determine the redemption value of the U.S. T-Bill.

(A) 997

(B) 999

(C) 1001

(D) 1003

(E) 1005

C: $P\left(1+.1\left(\frac{180}{365}\right)\right)=1000 \Rightarrow P=953.00$

US: $R\cdot\left(1-.1\left(\frac{180}{360}\right)\right)=953.00$

$\Rightarrow R=1003.16$

2. Given $\delta_t=\frac{0.5t}{2+t^2}$ determine the semiannual effective discount rate for last half of the first year.

(A) 0.069

(B) 0.071

(C) 0.073

(D) 0.075

(E) 0.077

$\delta_t=\frac{1}{4}\cdot\frac{2t}{2+t^2} \Rightarrow a(t)=\left(\frac{2+t^2}{2}\right)^{1/4}$

The last half of the 1st year is from $t=0.5$ to $t=1$.

$d_{[0.5,1]}=$ sedr from $t=0.5$ to $t=1$

$sdf_1^{0.5}=\frac{a(0.5)}{a(1)}=\left(1-d_{[0.5,1]}\right)$

$a(0.5)=\left(\frac{2.25}{2}\right)^{1/4}=1.0298\ldots$ [1]

$a(1)=\left(\frac{3}{2}\right)^{1/4}=1.1066\ldots$ [2]

$\therefore \frac{[1]}{[2]}=\left(1-d_{[0.5,1]}\right) \Rightarrow d_{[0.5,1]}=0.069\ldots$

3. Account A credits interest using a simple discount rate of 5%. Account B credits interest using a quarterly effective interest rate of i. At time $t = 4$, the forces of interest in the two accounts are equal. Determine the amount that needs to be invested into account B in order for the deposit to accumulate to 1000 after a three year period.

(A) 830

(B) 845

(C) 855

(D) 870

(E) 880

A: $\delta_t = \frac{.05}{1-.05t}$

B: $\delta_t = \ln(aaf) = \ln(1+i)^4 = 4\cdot\ln(1+i)$

At $t=4$: $\frac{.05}{1-.05(4)} = 4\cdot\ln(1+i) \Rightarrow \ln(1+i) = 0.015625$

$\Rightarrow 1+i = e^{0.015625} = qaf$

We need to discount 1000 for 3 years (12 quarters) in Account B:

$\therefore X = 1000(qdf)^{12} = 1000\left(e^{-0.015625}\right)^{12} = 829.03$

4. Jameis deposits 1000 into an account that credits interest using an interest rate of i, compounded semiannually. Two years after the deposit, Jameis withdraws 500 from the account. Two years after the withdrawal, Jameis has 750 in the account. Determine i.

$\frac{i}{2} = seir$

$1+\frac{i}{2} = saf$

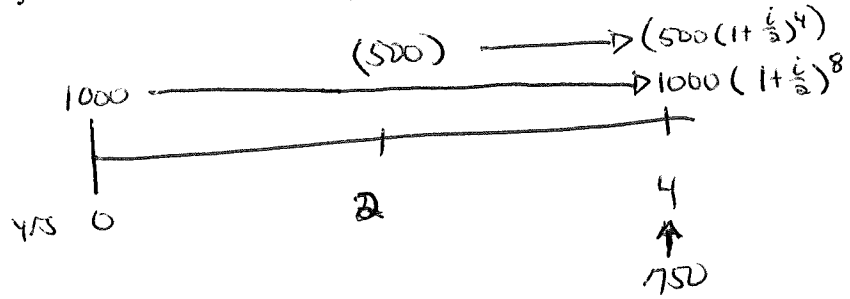
(A) 3.59%

(B) 3.65%

(C) 7.17%

(D) 7.30%

(E) 7.60%



$\therefore 1000\left(1+\frac{i}{2}\right)^8 - 500\left(1+\frac{i}{2}\right)^4 = 750$

quadratic in $\left(1+\frac{i}{2}\right)^4$

$a = 1000$

$b = -500$

$c = -750$

$\therefore \left(1+\frac{i}{2}\right)^4 = \frac{500 \pm \sqrt{500^2 - 4(1000)(-750)}}{2000} = 1.1513\ldots$

$\Rightarrow i = 7.174\ldots\%$

5. You are given a loan on which interest is charged over a 4-year period as follows:

I. An annual effective rate of discount of 6% for the first year
II. A nominal rate of discount of 5% compounded every 2 years for the second year
III. A nominal rate of interest of 5% compounded semiannually for the third year
IV. A force of interest of 5% for the fourth year

Determine the equivalent annual effective rate of interest over the 4-year period.

$= i \Rightarrow 1+i = aaf$

(A) 5.00%

(B) 5.25%

$paf_0^4 = (1+i)^4 = (1-.06)^{-1} \cdot \left(1-\frac{.05}{1/2}\right)^{-1/2} \cdot \left(1+\frac{.05}{2}\right)^2 \cdot e^{.05}$

(C) 5.50%

(D) 5.75%

$\Rightarrow i = 0.05494\ldots$

(E) 6.00%

6. Determine which of the following equations represents the correct relationship between a nominal interest rate compounded monthly and a nominal interest rate compounded quarterly.

(A) $i^{(4)} = 4\left[\left(1+\frac{i^{(12)}}{12}\right)^4 - 1\right]$

$\frac{i^{(4)}}{4} = qeir \Rightarrow 1+\frac{i^{(4)}}{4} = qaf$

(B) $i^{(4)} = 4\left[\left(1+\frac{i^{(12)}}{12}\right)^{12} + 1\right]$

$\frac{i^{(12)}}{12} = meir \Rightarrow 1+\frac{i^{(12)}}{12} = maf$

(C) $i^{(4)} = 4\left[\left(1+\frac{i^{(12)}}{12}\right)^4 + 1\right]$

$qaf = (maf)^3$

(D) $i^{(4)} = 4\left[\left(1+\frac{i^{(12)}}{12}\right)^3 - 1\right]$

$\Rightarrow 1+\frac{i^{(4)}}{4} = \left(1+\frac{i^{(12)}}{12}\right)^3$

(E) $i^{(4)} = 4\left[\left(1+\frac{i^{(12)}}{12}\right)^{12} - 1\right]$

$\Rightarrow i^{(4)} = 4\left[\left(1+\frac{i^{(12)}}{12}\right)^3 - 1\right]$

7. Determine the force of interest at time $t = 3$ for an account for which the amount function is $a(t) = 8t^2 + 32t + 100$.

(A) 0.10

(B) 0.15

(C) 0.20

(D) 0.25

(E) 0.30

$a'(t) = 16t + 32$

$\Rightarrow \delta_t = \dfrac{16t+32}{8t^2+32t+100}$

$\therefore \delta_3 = \dfrac{16(3)+32}{8(3^2)+32(3)+100} = \dfrac{80}{268} = 0.2985\ldots$

8. In order to pay off a debt, Tom makes a second payment of 1000 exactly three years after his first payment of 2000. Using an annual effective discount rate of d, the total present value of the two payments, three years before his first payment, is 1989.44. Determine the corresponding monthly accumulation factor (maf) for Tom's debt.

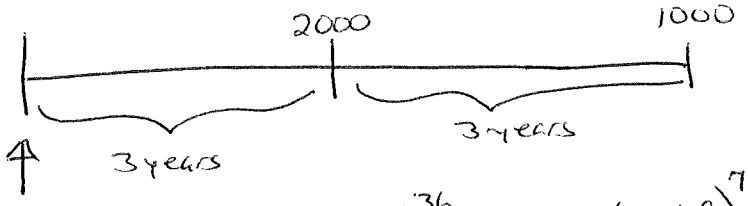
(A) 1.007

(B) 1.009

(C) 1.011

(D) 1.013

(E) 1.015



$1989.44 = 2000(mdf)^{36} + 1000(mdf)^{72}$

quadratic in $(mdf)^{36}$

$a = 1000$
$b = 2000$
$c = -1989.44$

$\therefore (mdf)^{36} = \dfrac{-2000 \pm \sqrt{2000^2 - 4(1000)(-1989.44)}}{2(1000)} = 0.7289\ldots$

$\Rightarrow mdf = 0.99125\ldots$

$\Rightarrow maf = 1.0088\ldots$

9. An account credits interest using a simple interest rate i for the first half of the first year. Thereafter, interest is credited using a nominal interest rate of i, compounded semiannually. A deposit of 5000 at time $t = 0$ accumulates to 5370 after 3 years. Determine i.

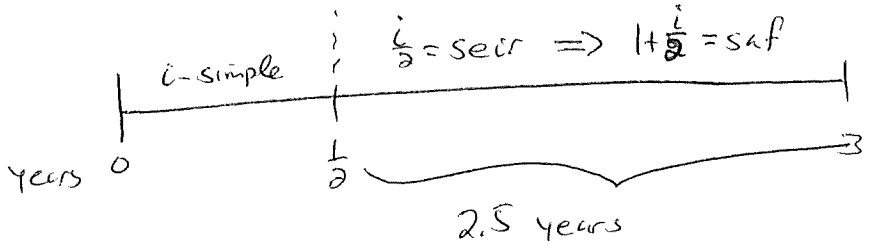
(A) 1.2%

(B) 2.4%

(C) 3.6%

(D) 4.8%

(E) 6.0%



$$\therefore 5000\left(1+i\cdot\tfrac{1}{2}\right)\left(1+\tfrac{i}{2}\right)^5 = 5370$$

$$\Rightarrow 5000\left(1+\tfrac{i}{2}\right)^6 = 5370$$

$$\Rightarrow i = 0.0239\ldots$$

10. An account credits interest using a simple interest rate of 4%. A deposit at time $t = 0$ accumulates to 1200 at time $t = 5$. Determine the amount the deposit had accumulated to at time $t = 2$.

$a(t) = 1 + .04t$

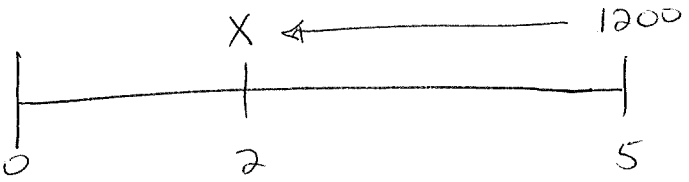
(A) 1040

(B) 1050

(C) 1060

(D) 1070

(E) 1080



$$X = 1200\,\frac{a(2)}{a(5)} = 1200\,\frac{1+.04(2)}{1+.04(5)}$$

$$\therefore X = 1080$$